GCE

Rewarding Learning

Revised GCE

Mathematics

Assessment Unit A2 2
assessing
Applied Mathematics

Practice Paper and Mark Scheme

For first teaching from September 2018
For first award of AS Level in Summer 2019
For first award of A Level in Summer 2019

Centre Number

Candidate Number

ADVANCED SUBSIDIARY (AS)
General Certificate of Education

Mathematics

Assessment Unit A2 2
Assessing
Applied Mathematics
[AMT21]

PRACTICE PAPER

TIME
1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
Candidates must answer **all** questions from sections A and B.
Equal time should be spent on each section.
Show clearly the full development of your answers. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.
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INFORMATION FOR CANDIDATES
The total mark for this paper is 100. The total available mark for each section of this paper is 35.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8 ms^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is ln z where it is noted that $\ln z \equiv \log_e z$

Answer all questions

SECTION A

Mechanics

1. A ball of mass $3m$ kg is travelling horizontally in a straight line with a velocity of 2ms^{-1}
A bat strikes the ball exerting an impulse that directly opposes the motion of the ball.
The ball then moves with a speed of 4ms^{-1} in the opposite direction.

(i) Find, in terms of m, the magnitude of the impulse exerted on the ball by the bat. [3]

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(ii) If the bat applies an average force of 30N and is in contact with the ball for 0.012 seconds find the mass of the ball. [2]

2. A driver of a car suddenly observes a dog running out in front of her car and applies the brakes immediately.

The acceleration , a, of the car after t seconds is given by

$$a = \left(\frac{5}{6}t - 5\right) \text{ms}^{-2} \qquad \text{for } 0 \leq t \leq 6$$

The car comes to rest after 6 seconds.

(i) Find an expression for the velocity of the car after t seconds. [4]

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The dog is 31 m from the car when the brakes are applied.

(ii) Assuming that the dog remains directly in the path of the car, find how far the car is from the dog when the car stops. [5]

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3. A basketball basket is a height of 3 metres above the horizontal floor.

John is standing a horizontal distance of 3.2m from the basket.

He throws a basketball from a height of 2.2m above the floor with a speed of u ms^{-1} inclined at $\alpha = \sin^{-1}(\frac{3}{5})$ to the horizontal.

The time taken for the ball to reach the basket is t seconds.

(i) Show that $ut = 4$ [3]

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(ii) Find t. [4]

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(iii) Find the angle the velocity of the ball makes with the vertical when it reaches the basket. [6]

4. At time t seconds a particle P has a vector displacement from a fixed origin O as follows

$$\mathbf{r} = [\tfrac{2}{3}t^3\mathbf{i} + \left(\tfrac{3}{2}t^2 - 3\right)\mathbf{j}] \quad \text{metres}$$

Find the displacement vector of P from O when P has a speed of 10ms^{-1} [8]

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5. A uniform ladder AB, of mass 40 kg rests with end A on rough, horizontal ground and end B against a rough, vertical wall.

End A is 1.2m from the wall and end B is 4m above the ground. Two of the external forces acting on the ladder are shown in **Fig 1** below.

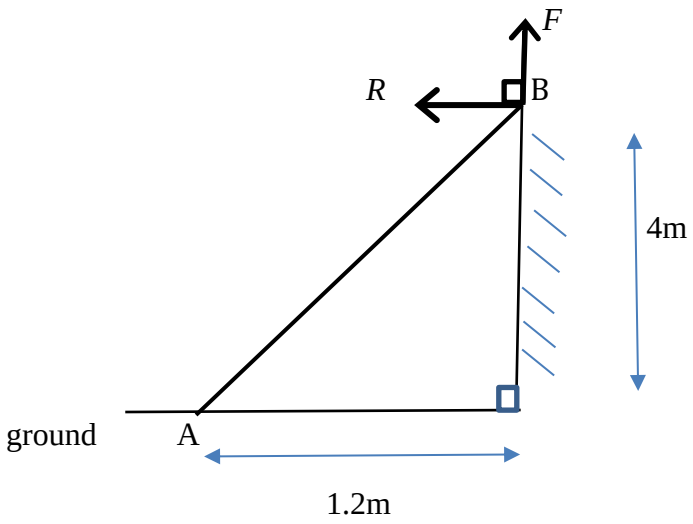
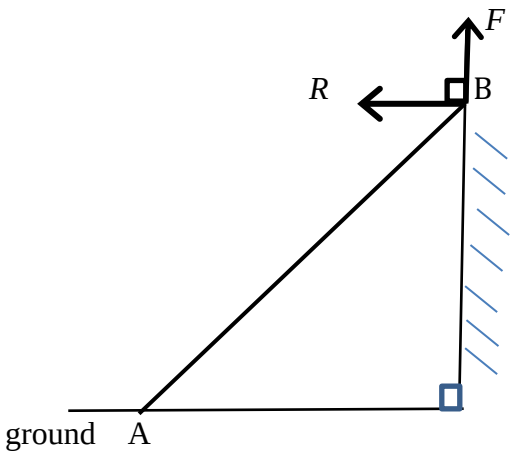


Fig 1

(i) Complete the force diagram below. [2]



(ii) By taking moments of about A show that

$$10R + 3F = 60g$$ [5]

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The coefficient of friction between the ladder and the wall is 0.4

The system is in limiting equilibrium.

(iii) Find the value of F. [3]

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(iv) Find the coefficient of friction between the ladder and the floor. [5]

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SECTION B

Statistics

6. Twenty teams play in the Premier football league.

Fig.2 below shows a scatter graph which represents the "goals for" and the "goals against" for each of these teams in a season.

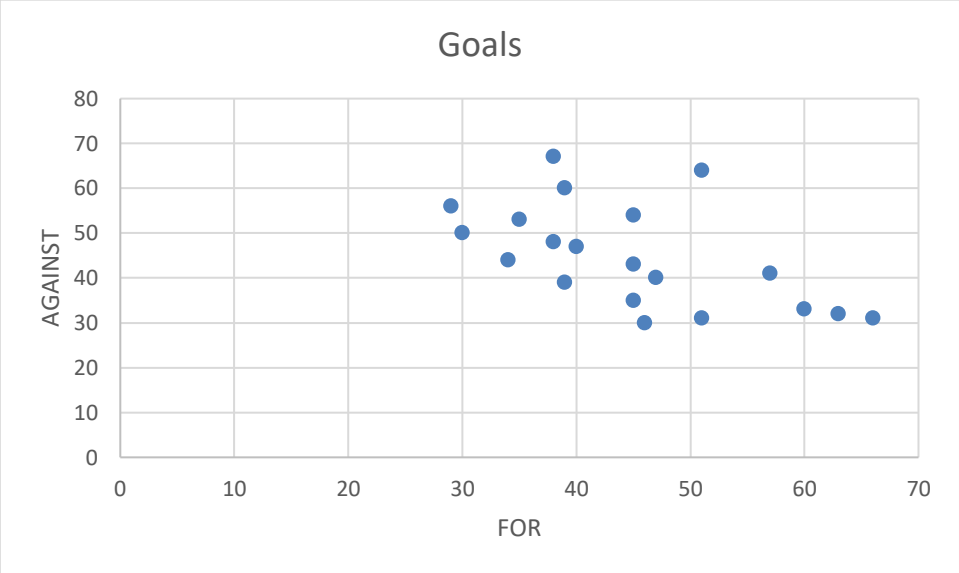

Fig.2

The Product Moment Correlation Coefficient r is found to have a value of -0.572

(i) Why would you expect the product moment correlation coefficient to be negative? [2]

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(ii) Using the tables provided, test this value of r at a 1% level of significance. [3]

7. The lengths of leaves on a large tree are Normally distributed with mean μ mm and standard deviation 6mm.

18 percent of leaves have lengths greater than 155.5 mm.

(i) Find the value of μ. [4]

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(ii) Find the probability that a randomly selected leaf has a length less than 160 mm. [4]

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(iii) Given that a leaf selected at random has a length less than 160 mm in length, find the probability that the leaf has a length between 155.5 mm and 160 mm. [5]

8. Customers at a supermarket have been suspecting that the length of tinfoil being sold is less than 10 m as stated on the box.

The manager takes a sample of 50 boxes and measures the length of tinfoil they contain.
The average length is 999.4 cm, and the sample standard deviation is 2 cm.

(i) Assuming Normality of the population distribution, carry out a test at the 5% significance level to decide whether the suspicions of the customers are correct. [8]

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The manager then tests his hypothesis at the 1% significance level to decide whether the suspicions of the customers are correct.

(ii) Would this have affected the outcome of his conclusion? [2]

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9. The manager of a basketball team believes they score on average 80% of their penalty shots taken in matches.

However, the coach believes their form is better than that and he decides to test his hypothesis at the 5% level of significance.

The coach keeps a record of penalty shots taken over a run of matches and notes that they scored 46 out of 50 shots attempted.

(i) Carry out the hypothesis test to test if the coach's belief is correct. [8]

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(ii) What is the critical value for the number of successful shots at this level of significance? [3]

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10. Two DVD rental companies, company 1 and company 2, each carry out a survey off their clients.

(i) Company 1 knows that:

20% of its customers rent DVDs on a Monday,

35% of its customers rent DVDs on a Friday,

45% of its customers rent DVDs on a Monday or a Friday.

Using a Venn diagram, or otherwise, find the probability p that a customer chosen at random from this group of customers rent DVDs on both days. [6]

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(ii) Company 2 knows that:

60% of their membership is male,

70% of their male membership rented DVDs more than 5 times a month,

55% of their female membership rented DVDs more than 5 times a month.

In a particular month, a randomly selected member was found to have rented DVDs 6 times.

Find the probability that the member was female. [5]

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THIS IS THE END OF THE PAPER

ADVANCED

General Certificate of Education

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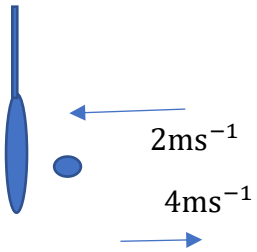
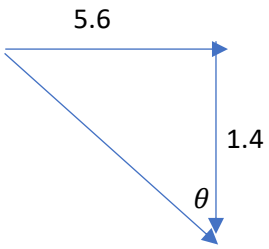
[AMT21]

PRACTICE PAPER

MARK SCHEME

		AVAILABLE MARKS
1.(i)		
		
$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	
$\mathbf{I} = 3m(4) - 3m(-2)$	W1	
$\lvert\mathbf{I}\rvert = 18m$ Ns	W1	
(ii) $\mathbf{I} = \mathbf{F}t$	M1	
$18m = 30 \times 0.012$		
$m = 0.02\text{kg}$	W1	5

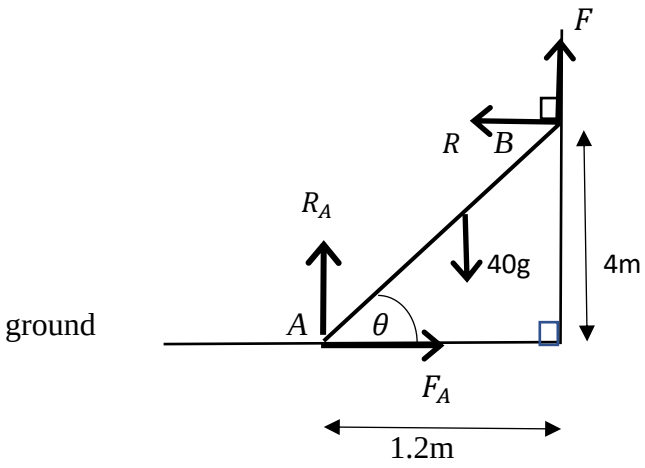
		AVAILABLE MARKS
2(i)		
$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{5}{6}t - 5$		
$v = \frac{5}{12}t^2 - 5t + c$	M1W2	
If $t = 6$ then $v = 0$		
then $0 = \frac{5}{12}6^2 - 5(6) + c$		
$0 = 15 - 30 + c$		
$c = 15$	MW1	
$v = \frac{5}{12}t^2 - 5t + 15\ \mathrm{ms}^{-1}$		
(ii) $v = \frac{5}{12}t^2 - 5t + 15$		
$s = \frac{5}{36}t^3 - \frac{5}{2}t^2 + 15t + d$	M1W3	
If $t = 0$ $s = 0 \Rightarrow d = 0$		
$s = \frac{5}{36}t^3 - \frac{5}{2}t^2 + 15t$		
If $t = 6$ $s = \frac{5}{36}(6)^3 - \frac{5}{2}(6)^2 + 15(6)$		
$s = 30 - 90 + 90 = 30$		
The car stops 1m from the dog.	MW1	9

		AVAILABLE MARKS
3.		
(i) $s_x = u\cos\alpha\, t$	M1W1	
$3.2 = ut\frac{4}{5}$		
$ut = 4$	W1	
(ii) $s_y = u\sin\alpha\, t - \frac{1}{2}gt^2$	M1	
$0.8 = 4(0.6) - \frac{1}{2}(9.8)t^2$	MW2	
$t^2 = \frac{16}{49}$		
$t = \frac{4}{7}$ s	W1	
(iii) $ut = 4$		
$u\frac{4}{7} = 4 \implies u = 7$	MW1	
$v_y = 7\left(\frac{3}{5}\right) - \frac{4}{7}$ g		
$v_y = 4.2 - 5.6 = -1.4$	M1W1	
$v_x = 7\left(\frac{4}{5}\right) = 5.6$	W1	
	MW1	
$\theta = \tan^{-1} 4 = 76.0°$ (3sf)	MW1	13

		AVAILABLE MARKS
4. $\mathbf{r} = [\frac{2}{3}t^3\mathbf{i} + \left(\frac{3}{2}t^2 - 3\right)\mathbf{j}]$ metres		
$\mathbf{v} = [\,2t^2\mathbf{i} + 3t\mathbf{j}]$	M1W2	
Speed $= \lvert\mathbf{v}\rvert = \sqrt{(2t^2)^2 + (3t)^2}$	M1W1	
$= \sqrt{4t^4 + 9t^2}$		
Hence $4t^4 + 9t^2 = 100$	M1	
$4t^4 + 9t^2 - 100 = 0$		
$(4t^2 + 25)(t^2 - 4) = 0$		
$t^2 \neq -\frac{25}{4},\ \ t^2 = 4$		
Hence $t = 2$	W1	
Hence $\mathbf{r} = \frac{16}{3}\mathbf{i} + 3\mathbf{j}$ m	MW1	8

AVAILABLE MARKS

5.



MW2

(ii) $M(A)$ $40\text{g} \times 0.6 = 4R + 1.2F$ M1M1W3

$24\text{g} = 4R + 1.2F$

$240\text{g} = 40R + 12F$

$60\text{g} = 10R + 3F$ (1)

(iii) $F = 0.4R$ MW1

$\Longrightarrow \quad R = 2.5F$

Combining with (1)

$10R = 60\text{g} - 3F = 25F$ M1

$28F = 60\text{g}$

$F = \frac{15}{7} \times \text{g} = 21\text{N}$ W1

(iv) If $F = 21$ N then

		AVAILABLE MARKS
$10R = 60\text{g} - 3(21)$		
$R = 58.8 - 6.3 = 52.5$ N	MW1	
Resolve horizontally $R = F_A$ hence $F_A = 52.5$ N	MW1	
Resolve vertically $21 + R_A = 40\text{g}$	M1	
$R_A = 392 - 21 = 371$	W1	
Hence $F_A = \mu R_A$		
$52.5 = \mu 371$		
$\mu = 0.142$	MW1	15

	AVAILABLE MARKS

6. (i) If the team is playing well you will tend to score more goals and let less in.

If the team is not playing well, you will tend to score less goals and let more in.

Hence you would expect r to be negative. MW2

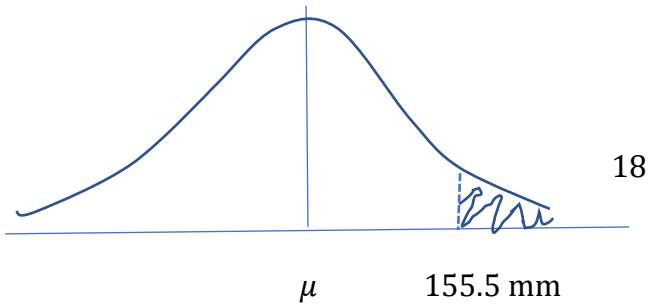
(ii) $n = 20,\ r = -0.572$ MW1

To be significant the 1% level of significance $|r| \geq 0.5155$ MW1

$|r| = 0.572 \geq 0.5155$

Hence r is significant at the 1% level of significance MW1 5

7. (i) $\sigma = \text{SD} = 6$ mm



$\emptyset(z) = 0.82$ M1

$z = 0.9153645 \ldots$ MW1

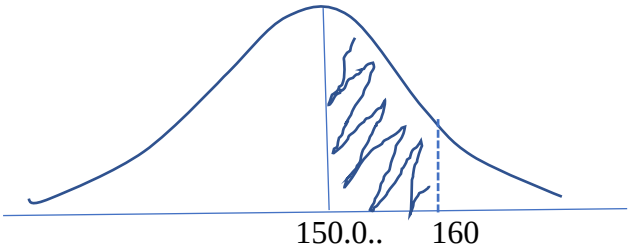
$\frac{155.5-\mu}{6} = 0.9153645..$ M1

$\mu = 155.5 - 6(0.9153645)$

$= 150.01150 = 150$ W1

	AVAILABLE MARKS

(ii) $X \sim N(150, 6^2)$



MW2

$\mu = 150.0115$ and $\sigma = 6$mm

Calculator use:

Probability represented by shaded area = 0.452019 M1

Probability length less than 160 mm = 0. 952019.. = 0.952 W1

Solution not using calculator with normal distribution function

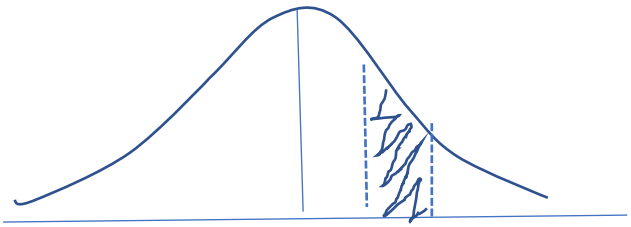
$$X \sim \mathrm{N}(150.0115, 6^2)$$

$$\mathrm{P}(X < 160) = \mathrm{P}\left(Z < \frac{160 - 150.0115}{6}\right) \quad \text{M1W1}$$

$$= \mathrm{P}(Z < 1.66475) \quad \text{W1}$$

Probability =0.952018… = 0.952 W1

(iii)



MW1

Calculator use:

Cumulative probability function between 155.5 and 160 with

$\mu = 150.0115$ and $\sigma = 6$

$\mathrm{P}(155.5 \leq X < 160) = 0.132180\ldots\ldots$ = 0.132 MW1

Solution not using calculator with normal distribution function.

AVAILABLE MARKS

$$X \sim \mathrm{N}(150.0115, 6^2)$$

$$\mathrm{P}(155.5 \leq X < 160) = \mathrm{P}(X < 160) - \mathrm{P}(X < 155.5) \qquad \text{MW1}$$

$$= 0.952019 - 0.82$$
$$= 0.132019 = 0.132 \qquad \text{MW1}$$

Probability that given the length of leaf lies between 155.5 and 160 given it is less than 160mm

$$\frac{0.1312019\ldots}{0.952019} = 0.13867 = 0.139 \qquad \text{M1W1W1}$$

13

AVAILABLE MARKS

8(i)

$H_0: \mu_0 = 1000\text{cm}$

$H_1: \mu_0 < 1000\text{cm}$ MW1

One tailed test at the 5% level of significance.

Reject H_0 if $z < -1.645$ MW1

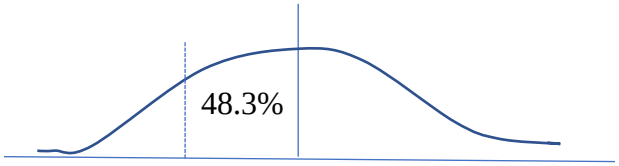
$\bar{X} \sim N(50, 2^2)$ M1

$$z_{\text{test}} = \frac{999.4 - 1000}{\frac{2}{\sqrt{50}}} = -2.12132$$ M1W1W1

Since $z_{\text{test}} < -1.645$ we reject H_0. There is sufficient evidence at the 5% level of significance to suggest that the length of tinfoil being sold is less than the 10m stated on the box. MW2

Calculator Use: Last 5 Marks

Probability cumulative distribution function. $\mu = 1000,\ \sigma = \frac{2}{\sqrt{50}}$



MW1

999.4 1000

Probability = 0. 4830525.. = 48.3% M1

Probability below 999.4 is 1.695% W1

Since $1.695\% < 5\%$ we reject H_0. There is sufficient evidence at the 5% level of significance to suggest that the length of tinfoil being sold is less than the 10m stated on the box. MW2

(ii)

If testing at the 1% significance level, since $-2.12132 > -2.3263$, we do not reject H_0.There is not sufficient evidence at the 1% level of significance to suggest that the length of tinfoil being sold is less than the 10m stated on the box. MW2

Calculator Use:

If testing at the 1% significance level, since $1.695\% > 1\%$, we do not reject H_0. There is not sufficient evidence at the 1% level of significance to suggest that the length of tinfoil being sold is less than the 10m stated on the box. MW2

AVAILABLE MARKS
10

	AVAILABLE MARKS

9. (i)

$X \sim \text{Bin}(50, 0.8)$ MW1

$\text{H}_0: p = 0.8$ MW1

$\text{H}_1: p > 0.8$ MW1

$\text{P}(X \geq 46) = 1 - \text{P}(X \leq 45)$ M1 MW1

$= 1 - 0.981503.. = 0.018497 = 1.85\%$ W1

There is sufficient evidence to reject at the 5% level of significance the manager's belief that his team score on average 80% of their penalties. MW2

(ii)

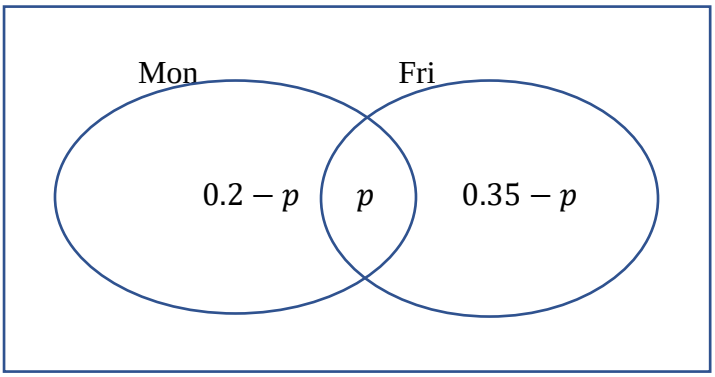
x	43	44	45	46
$\text{P}(X \leq x)$	0.8966017725	0.95197	0.981503	0.9943436

$\text{P}(X \geq 46) = 1 - \text{P}(X \leq 45) = 0.018497 = 1.85\%$ M1

$\text{P}(X \geq 45) = 1 - \text{P}(X \leq 44) = 0.04803 = 4.80\%$

$\text{P}(X \geq 44) = 1 - \text{P}(X \leq 43) = 0.103398.. = 10.3\%$ W1

Critical value for the number of successful penalty shots at the 5% significance level is 45 MW1 11

		AVAILABLE MARKS
10.(i) Using the Venn diagram given below		
	MW3	
$0.2-p+p+0.35-p=0.45$	M1W1	
$p=0.10$	W1	
(ii) P(female|rented more than 5 times a month)		
$=\frac{\text{P(female and rented more than 5 times a month)}}{\text{P(rented more than 5 times a month)}}$	M1	
P(male and rented more than 5 times a month) $=0.60\times0.70=0.42$	MW1	
P(female and rented more than 5 times a month) $=0.40\times0.55=0.22$	MW1	
P(rented more than 5 times a month) $=0.42+0.22$	MW1	
$=\frac{0.22}{(0.22+0.42)}=\frac{11}{32}=0.34375=0.344$ (3sf)	W1	11
	Total	**100**

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