Summer 2021

Rewarding Learning

Summer 2021
GCSE Further Mathematics Support
Unit 1: Pure Mathematics
Question and Answer Booklet

Version 1.0

CONTENTS

Introduction

This booklet comprises 110 Pure Mathematics questions written for the CCEA GCSE Further Mathematics specification by the Examining Team. The questions are spread across every topic in Unit 1 Pure Mathematics and are in the familiar style of the examination papers. There is a range of difficulty, from basic through to challenging.

Answers are provided for every question. Questions marked with an asterisk have more extensive worked solutions provided in the answer section. As a guide, total marks have been provided for some of the questions.

Most of these questions have been subjected to a revision process, but not all. A full paper revising process, however, has not been applied and the questions have not gone through CCEA's normal quality assurance process for question paper production.

Please note, this booklet of questions is an optional support resource and sits separate to the CCEA 2021 Assessment Resource. Teachers can use the booklet as a classroom resource or students can use the booklet as a revision resource. It can also act as an additional resource for those working remotely.

Here then are the contents of several Pure Unit 1 examinations for use by teachers and students alike.

March 2021

1.1 Algebra - Algebraic Fractions

Q.1.1.1 (*) Simplify the algebraic expressions:

(i) $\frac{5}{x^2-x} + \frac{2}{x^2-4x+3}$

(ii) $\frac{x^2-4}{x^2-x-6} \div \frac{6}{3x-9}$

Q.1.1.2 **(i)** Show that $\frac{2x+3}{x-3} - \frac{x-2}{4-x}$ can be written as $\frac{3x^2-10x-6}{x^2-7x+12}$

(ii) **Hence**, or otherwise, solve the equation

$$\frac{2x+3}{x-3} - \frac{x-2}{4-x} = 7$$

Q.1.1.3 (*) (i) Show that $\frac{2x-1}{x-2} - \frac{x-3}{6-x}$ can be written as $\frac{3(x^2-6x+4)}{x^2-8x+12}$

(ii) **Hence**, or otherwise, solve the equation

$$\frac{2x-1}{x-2} - \frac{x-3}{6-x} = 3$$

Q.1.1.4 (a) Express $\frac{x-1}{x+5} + \frac{x+4}{2x+3}$ as a single fraction [4]

(b) Simplify the expression $\frac{x^2+5x+4}{3} \times \frac{2}{x^2-16}$ [3]

Q.1.1.5

Simplify the expressions (a) $\frac{x^2-3x+2}{x^2-5x+6} \times \frac{x^2+2x-3}{x^2-9}$ [4]

(b) $\frac{x^2-3x+2}{x^2-5x+6} \div \frac{x^2+2x-3}{x^2-9}$ [3]

Q.1.1.6 Simplify the expression $\frac{2x-1}{x+2} - \frac{x}{2x+3}$ [4]

Q.1.1.7

Simplify the expression $\frac{x+1}{9-x^2} \times \frac{3-x}{x^2-10x-11}$ [3]

1.2 Algebra - Algebraic Manipulation

Q.1.2.1 (*) Expand $(3x + 2)(2x - 1)(x + 1)$ [3]

Q.1.2.2 Multiply out $(x - 4)(x + 2)(x - 1)$ [3]

Q.1.2.3 Expand $(x - 5)(x - 7)(x + 1)$ [3]

Q.1.2.4 Multiply out $(x - 3)(3x + 1)(2x - 1)$ [3]

Q.1.2.5 Expand $(x + 5)(2x + 1)(x - 1)$ [3]

Q.1.2.6 Multiply out $(3x - 1)(3x + 1)(x + 1)$ [3]

1.3 Algebra - Completing the Square

Q.1.3.1 (*) Solve the equation

$$x^2 - 8x - 7 = 0$$

by **completing the square**.

Give your answer in the form $a \pm \sqrt{b}$, where a and b are integers.

Q.1.3.2 (*) A curve is defined by the equation $y = x^2 - 7x + 3$

(i) By **completing the square**, write the equation in the form $y = (x - a)^2 - b$, where a and b are constants.

(ii) **Hence** write down

(a) the minimum value of the curve,

(b) the value of x at this minimum value.

Q.1.3.3 Solve the equation

$$x^2 + 6x - 7 = 0$$

by **completing the square**.

Q.1.3.4 Find the values of x where the curve

$$y = x^2 - 6x + 1$$

cross the straight line $y = 4$ by using the method of **completing the square**.

Q.1.3.5

Solve the equation:

$$x^2 = 12x + 7$$

by completing the quare, giving your answer in the form $a \pm \sqrt{b}$ where a and b are whole numbers.

[4]

Q.1.3.6

Solve the equation:

$$x^2 + 9 = 13 - x$$

completing the square, giving your answer in the form $a \pm \sqrt{b}$ where a and b are fractions [4]

1.4 Algebra - Simultaneous Equations

Q.1.4.1 (*) A store sells three different types of cereals - Economy, Standard and Deluxe.

A packet of Economy cereal costs £x, a packet of Standard cereal costs £y and a packet of Deluxe cereal costs £z.

One week the store sold 20 Economy, 30 Standard and 5 Deluxe packets of cereal. The total sales were £100

(i) Show that x, y and z satisfy the equation

$$4x + 6y + z = 20$$

In the following week the store sold 30 Economy, 36 Standard and 24 Deluxe packets of cereal. The total sales were £174

(ii) Show that x, y and z also satisfy the equation

$$5x + 6y + 4z = 29$$

The store had a sale in which the Standard cereal was sold at half price and a packet of Deluxe cereal was reduced by 50p. The Economy cereal stayed at the same price.

During the first week of the sale, the store sold 5 Economy, 40 Standard and 30 Deluxe packets of cereal. The total sales were £111

(iii) Show that x, y and z also satisfy the equation

$$5x + 20y + 30z = 126$$

(iv) Solve the equations

$$\begin{aligned} 4x + 6y + z &= 20 \\ 5x + 6y + 4z &= 29 \\ 5x + 20y + 30z &= 126 \end{aligned}$$

to find the values of x, y and z.

You must show clearly each stage of your solution.

In the second week of the sale, the store sold 16 Economy, 80 Standard and 32 Deluxe packets of cereal.

(v) Find the total sales for this week.

Q.1.4.2 Ten criminals were convicted in a court.

Five were convicted for assault and each sentenced to x years in prison.

Three were convicted for arson and each sentenced to y years in prison.

The remaining two were convicted for robbery and each sentenced to z years in prison.

The criminals were sentenced to a total of 74 years in prison.

(i) Write down an equation connecting x, y and z.

The sentence for assault was one year more than the sum of the sentences for arson and robbery.

(ii) Write down a second equation connecting x, y and z.

Upon appeal, one of those convicted for assault had his sentence reduced to 7 years, one of those convicted for arson had his sentenced reduced to 4 years, and one of those convicted for robbery had his sentence doubled.

The total sentence for the criminals was now 72 years.

(iii) Show that x, y and z also satisfy the equation

$$4x + 2y + 3z = 61$$

(iv) Find the original sentences imposed.

Q.1.4.3 Solve the simultaneous equations:

$$5x + 3y - 2z = 17$$

$$7x - 2y + 3z = 18.5$$

$$2x + 4y - 5z = 0.5$$

[8]

Q.1.4.4

Solve the simultaneous equations:

$$2x + 3y - 4z = -2.5$$
$$5x - 2y + 3z = 22.4$$
$$3x + 4y - 2z = 15$$

[8]

1.5 Algebra - Quadratic Inequalities

Q.1.5.1 (*) Solve the inequality

$$x^2 + 7x - 8 < 0$$

Q.1.5.2 (*) Solve the inequality

$$x^2 - 3x \geq 10$$

Q.1.5.3 Solve the inequality: [3]

$$2x^2 + 5x - 3 \geq 0$$

Q.1.5.4 Solve the inequality: [3]

$$3 + 11x - 4x^2 < 0$$

Q.1.5.5 Solve the inequality: [3]

$$x^2 - 7x + 6 < 0$$

Q.1.5.6 Solve the inequality: [3]

$$9x^2 - 16 \leq 0$$

2.1 Trigonometry

Q.2.1.1 (*) **(i)** Sketch the graph of $y = \cos x$ for $0° \leq x \leq 360°$.

(ii) Solve the equation

$$\cos x = -0.65$$

for $0° \leq x \leq 360°$.

(iii) **Hence** solve the equation

$$\cos (2\theta - 10°) = -0.65$$

for $0° \leq x \leq 180°$.

Q.2.1.2 (*) **(i)** Sketch the graph of $y = \sin x$ for $-180° \leq x \leq 180°$.

(ii) Solve the equation

$$\sin x = -0.45$$

for $-180° \leq x \leq 180°$.

(iii) **Hence** solve the equation

$$\sin (3\theta + 10°) = -0.45$$

for $-60° \leq x \leq 60°$.

Q.2.1.3 **(i)** Solve the equation

$$\cos x = -0.2$$

for $0° \leq x \leq 360°$.

(ii) **Hence** solve the equation

$$\sin (2\theta - 15°) = -0.2$$

for $0° \leq x \leq 180°$.

Q.2.1.4

(i) Solve the equation

$$\sin\theta = 0.3$$

for $0^o \leq \theta \leq 360^o$ [2]

(ii) Hence solve the equation

$$\sin(2x - 35^o) = 0.3$$

for $0^o \leq x \leq 180^o$ [3]

Q.2.1.5

(i) Solve the equation

$$\tan\theta = -0.6$$

for $-180^o \leq \theta \leq 180^o$ [2]

(ii) Hence solve the equation

$$\tan(\frac{x - 20^o}{2}) = -0.6$$

for $-360^o \leq x \leq 360^o$ [3]

3.1 Differentiation – Basic Operation

Q.3.1.1 (*) Find $\frac{dy}{dx}$ if $y = \frac{2}{7}x^{14} - \frac{14}{x^7}$

Q.3.1.2 (*) Find $\frac{d^2y}{dx^2}$ if $y = 8x^3 - 2 + \frac{4}{x^2}$

Q.3.1.3

If $y = 4x^6 - \frac{5}{x^2}$ find

(i) $\frac{dy}{dx}$ [2]

(ii) $\frac{d^2y}{dx^2}$ [2]

Q.3.1.4

If $y = 3x^2 - \frac{2}{x}$ find

(i) $\frac{dy}{dx}$ [2]

(ii) $\frac{d^2y}{dx^2}$ [2]

Q.3.1.5

If $y = x^8 - \frac{7}{x^5}$ find

(i) $\frac{dy}{dx}$ [2]

(ii) $\frac{d^2y}{dx^2}$ [2]

Q.3.1.6 (*) A curve is defined by

$$y = 4x^3 - 12x^2 + 21x + 2$$

Find the values of x for which $\frac{d^2y}{dx^2} = \frac{dy}{dx}$.

3.2 Differentiation - Finding Equations of Tangents and Normals at Points on a Curve

Q.3.2.1 (*) A curve is defined by the equation

$$y = 2x + \frac{3}{4x}$$

A point P on the curve has coordinates (-1, -2¾).

(i) Find the equation of the tangent to the curve at the point P.

(ii) Show that the normal to the curve at the point P is parallel to the straight line with equation

$$15y + 12x = 7$$

Q.3.2.2 (*) A curve is defined by the equation

$$y = ax^2 - 5x + b$$

where a and b are constants.

The equation of the tangent to this curve at the point (2, 6) is $y = 7x - 8$

Find the values of a and b.

Q.3.2.3 (*) A curve is defined by the equation

$$y = 7x - \frac{2}{x^2}$$

A point P on the curve has coordinates (-1, - 9).

(i) Find the equation of the tangent to the curve at the point P.

(ii) Find where the normal to the curve at the point P intersects the straight line with equation

$$y = 5 - 2x$$

Q.3.2.4 A curve is defined by the equation

$$y = 6 - 2x + x^2$$

(i) Find the equation of the tangent to this curve at the point (2, 6).

(ii) Find the coordinates of the point where this tangent meets the line

$$2y - 5x = 3$$

Q.3.2.5 A curve is defined by the equation

$$y = 2x^2 - 5x + 3$$

(i) Find the equation of the tangent to this curve at the point (2, 1).

(ii) Find the equation of the normal to this curve at the point (2, 1).

(iii) Find the coordinates of the other point where this normal cuts the curve.

Q.3.2.6 A curve is defined by the equation

$$y = 4x + 2x^2 - 5x^3$$

(i) Find the coordinates of the points on the curve where the tangents are parallel to the line

$$y = x - 2$$

(ii) Find the equation of the normal to this curve at the point (- 1, 3).

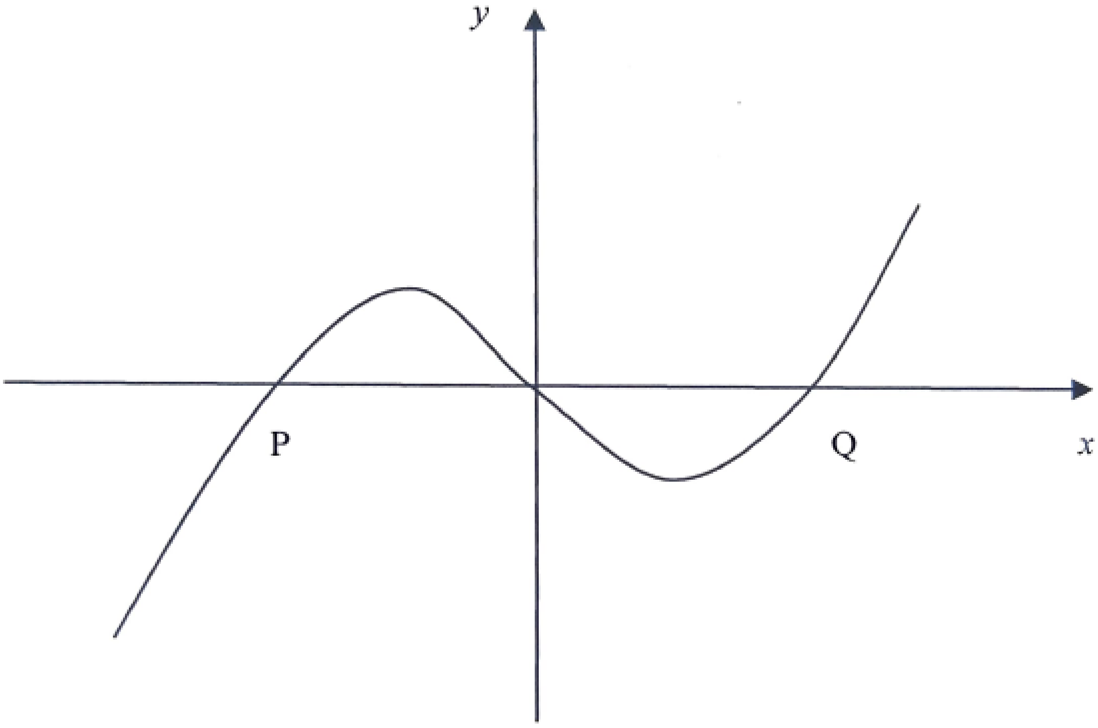
Q.3.2.7

A curve is defined by the equation $y = x^2 - 3x + 4$

(i) Find the equation of the normal to this curve at the point where x = 3 [5]

(ii) Find the coordinates of the other point where this normal meets the curve again [3]

Q.3.2.8
A sketch of the curve $y = x^3 - 4x$ is shown in the diagram below.



(i) Find the equation of the tangent at the point P. [5]

(ii) Find the equation of the normal at the point Q. [3]

(iii) Find the coordinates of the point at which the tangent at P meets the normal at Q. [3]

3.3 Differentiation - Simple Optimisation Problems

Q3.3.1
The profit £P from an investment after t months is given by the equation:

$$P = 400 + 2000t - 5t^2$$

(i) Find the maximum profit and after how many months this occurs. [4]

(ii) Prove that this is a maximum value. [2]

Q.3.3.2

The number of live cells C in a culture after t seconds is given by the equation:

$$C = 20{,}000t^4 - 270{,}000t + 500{,}000$$

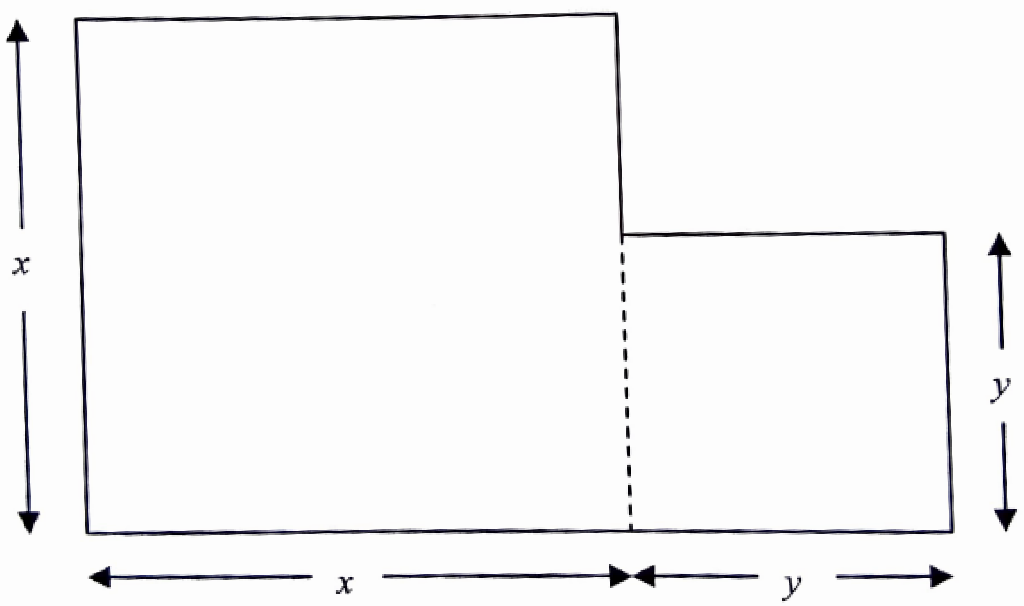
(i) Find a formula for the rate of change of the number of live cells per second. [2]

(ii) Find the minimum number of live cells and when this occurs. [3]

(iii) Prove that this is a minimum value. [2]

Q.3.3.3

A gardener wishes to dig two adjacent square plots of ground with sides x m and y m.
The total perimeter of the boundary is to be 22m.

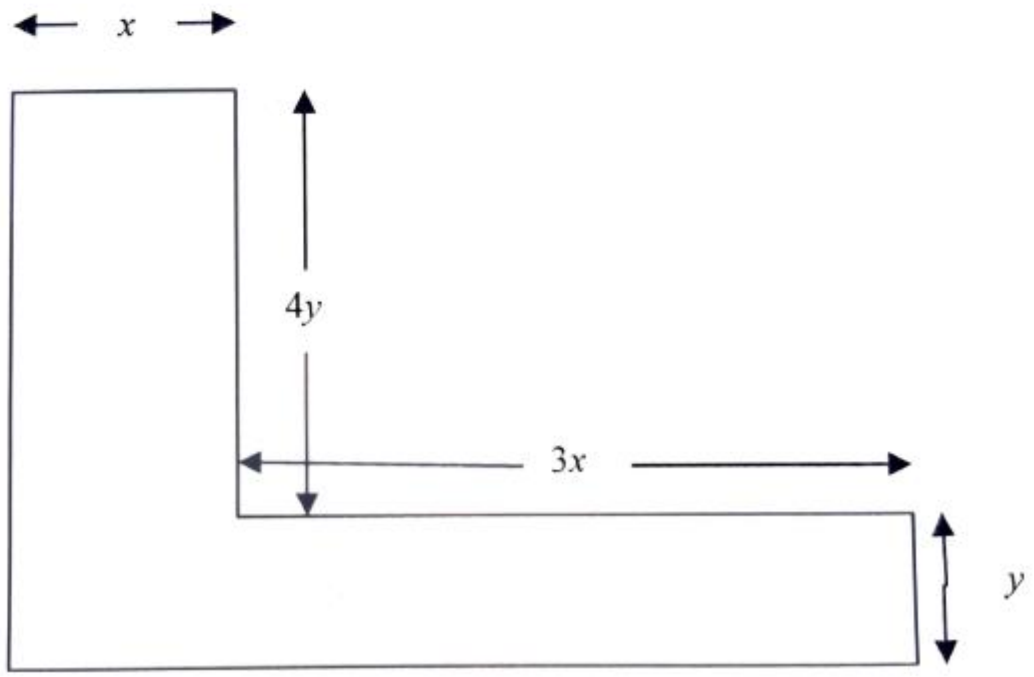


(i) Show that $y = 11 - 2x$ [1]

(ii) If the total area of both plots is to be 25 m^2, form a quadratic equation in x and hence find two pairs of solutions for x and y. [5]

(iii) If instead, the gardener wishes to minimise the total area of both plots, find the values of x and y which accomplish this. Prove that this area is a minimum. [5]

Q.3.3.4
A specialist cake maker wishes to make an L-shaped cake for a customer, with the dimensions of the base being shown in the diagram below:



The cake maker has 192cm of silver edging and he wishes to use all of this to attach around the entire perimeter of the cake.

(i) Show that $y = 19.2 - 0.8x$ [1]

(ii) Hence write down an expression for the area of the base in terms of x. [2]

(iii) Calculate the maximum area that the base of the cake may have, proving that it is a maximum. [5]

3.4 Differentiation - Elementary Curve Sketching of a Quadratic or Cubic Function

Q.3.4.1 (*) A curve is defined by the equation

$$y = 21 - 4x - x^2$$

(i) Find the coordinates of the points where the curve crosses the x-axis.

(ii) Find the coordinates of the turning point of the curve.

(iii) Using calculus, identify the turning point as either a maximum or a minimum point.

(iv) Sketch the graph of the curve $y = 21 - 4x - x^2$.

(v) Find the area bounded by this curve, the x-axis, the y-axis and a vertical line through the turning point.

Q.3.4.2 (*) A curve is defined by the equation

$$y = 2x(3 - x)(5 + 2x)$$

(i) Write down the coordinates of the points where the curve crosses the x-axis.

(ii) Find the coordinates of the turning points of the curve.

(iii) Using calculus, identify each turning point as either a maximum or a minimum point.

(iv) Sketch the graph of the curve $y = 2x(3 - x)(5 + 2x)$.

Q.3.4.3 (*) A curve is defined by the equation

$$y = 24x - 2x^2 - x^3$$

(i) Find the coordinates of the points where the curve crosses the x-axis.

(ii) Find the coordinates of the turning points of the curve.

(iii) Using calculus, identify each turning point as either a maximum or a minimum point.

(iv) Sketch the graph of the curve $y = 24x - 2x^2 - x^3$

Q.3.4.4

A curve is defined by the equation $y = x^3 + 11x^2 - 80x$

(i) Find the coordinates of the points where the curve crosses the x-axis. [3]

(ii) Find the coordinates of the turning points of the curve. [5]

(iii) Identify, using differentiation, whether each turning point is a maximum or minimum point. [2]

(iv) Hence sketch the curve on a blank pair of axes. [2]

(v) Find the area enclosed between the curve and the positive x-axis. [4]

Q.3.4.5

A curve is defined by the equation $y = x(x - 7)(x + 8)$

(i) Find the coordinates of the points where the curve crosses the x-axis. [2]

(ii) Find the coordinates of the turning points of the curve. [6]

(iii) Identify, using differentiation, whether each turning point is a maximum or minimum point. [2]

(iv) Hence sketch the curve on a blank pair of axes. [2]

(v) Find the area enclosed between the curve and the positive x-axis. [4]

Q.3.4.6

A curve is defined by the equation $y = x^3 + 4x^2 - 60x$

(i) Find the coordinates of the points where the curve crosses the x-axis. [3]

(ii) Find the coordinates of the turning points of the curve. [5]

(iii) Identify, using differentiation, whether each turning point is a maximum or minimum point. [2]

(iv) Hence sketch the curve on a blank pair of axes. [2]

(v) Find the area enclosed between the curve and the positive x-axis. [4]

Q.3.4.7

A curve is defined by the equation $y = x(x + 9)(x + 24)$

(i) Find the coordinates of the points where the curve crosses the x-axis. [2]

(ii) Find the coordinates of the turning points of the curve. [6]

(iii) Identify, using differentiation, whether each turning point is a maximum or minimum point. [2]

(iv) Hence sketch the curve on a blank pair of axes. [2]

Q.3.4.8

A curve is defined by the equation $y = x^3 + 39x^2 + 360x$

(i) Find the coordinates of the points where the curve crosses the x-axis. [3]

(ii) Find the coordinates of the turning point(s) of the curve. [5]

(iii) Identify, using differentiation, whether each turning point is a maximum or minimum point. [2]

(iv) Hence sketch the curve on a blank pair of axes. [2]

4.1 Integration – Indefinite Integration

Q.4.1.1 (*) Find $\int \left(3x^5 + \frac{1}{4x^3} - 1\right) \mathrm{d}x$ [3]

Q.4.1.2 Integrate [3]

$$\int \left(\frac{3}{x^3} - 4x^2\right) dx$$

Q.4.1.3 Find [3]]

$$\int \left(x^3 + \frac{2}{x^3}\right) dx$$

Q.4.1.4 Integrate [3]

$$\int (6 - x^4)dx$$

Q.4.1.5 Find [3]

$$\int \left(6x^2 - \frac{5}{7x^5}\right) dx$$

Q.4.1.6 Integrate [3]

$$\int \left(\frac{4}{5x^3} + 7x^4\right) dx$$

Q.4.1.7 (*) A function is defined by the equation

$$y = ax^2 + bx + c$$

where a, b and c are constants.

The derivative of the function is given by

$$\frac{dy}{dx} = 4x - 3$$

and when $x = 2$, $y = 6$

Express y in terms of x.

4.2 Integration - Form and Evaluate Definite Integrals

Q.4.2.1 Evaluate [3]

$$\int_{-1}^{1} (8x - 2x^2)dx$$

Q.4.2.2 Find [3]

$$\int_{0}^{3} (5 + 4x + 6x^2)dx$$

Q.4.2.3 Evaluate [3]

$$\int_{-2}^{3} (3x^2 - 4x^3)dx$$

Q.4.2.4 Find [3]

$$\int_1^4 (\frac{8}{x^3} - 5x^4)dx$$

4.3 Integration - Finding the Area Under a Curve

Q.4.3.1 (*) Find the area between the curve

$$y = 2x^2 - 5x - 3$$

and the x-axis.

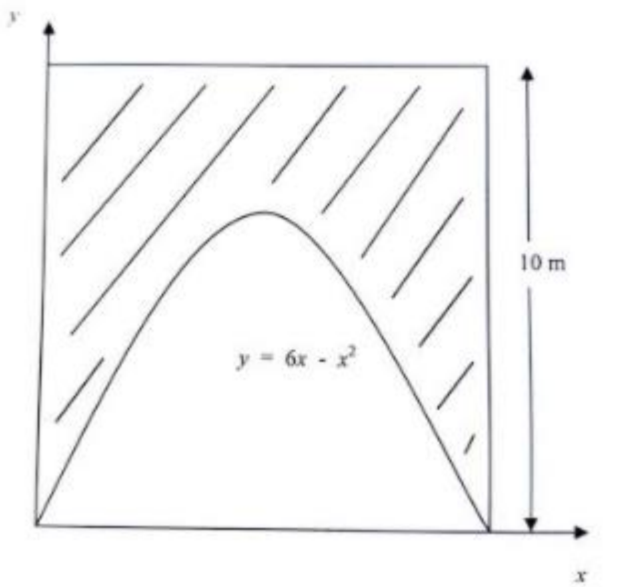
Q.4.3.2 (*) Find the area under the curve

$$y = 3x^2 + 4x + 1$$

between the lines $x = 1$, $x = 3$ and the x-axis.

Q.4.3.3

A cross-section of a stone bridge shown below can be modelled by the curve $y = 6x - x^2$ where x is in metres. It is surmounted by a rectangle 10m high.



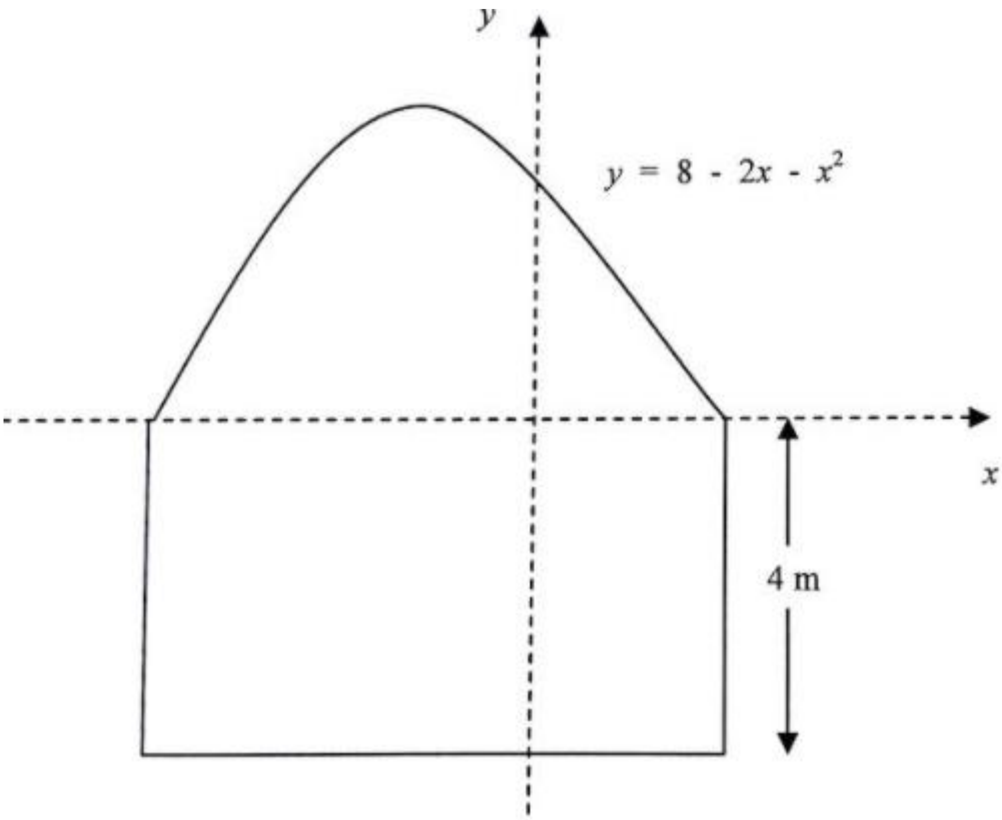
(i) Find the area under the curve and above the x-axis. [5]

(ii) Assuming the vertical sides of the bridge reach the x-axis at the same points as the curve, find the area of the cross-section of the stone bridge. [2]

Q.4.3.4

The diagram below shows the cross section of the tunnel.

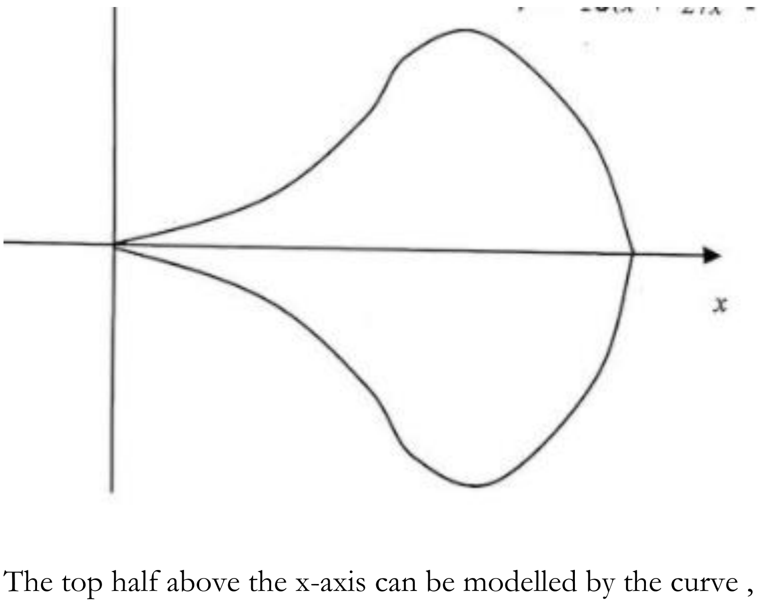
The roof can be modelled by the curve $y = 8 - 2x - x^2$ The height of the wall is 4m.



(i) Find the area between the curve and the x-axis. [5]

(ii) Hence, find the total area of the cross section of the tunnel. [2]

Q.4.3.5 A carpenter cuts a leaf shaped piece of wood, as shown in the diagram below.



The top half above the x-axis can be modelled by the curve ,

$$y = \frac{1}{10}(x + 27x^2 - 28x^3)$$

where x is in metres.

The piece of wood is symmetrical about the x-axis.

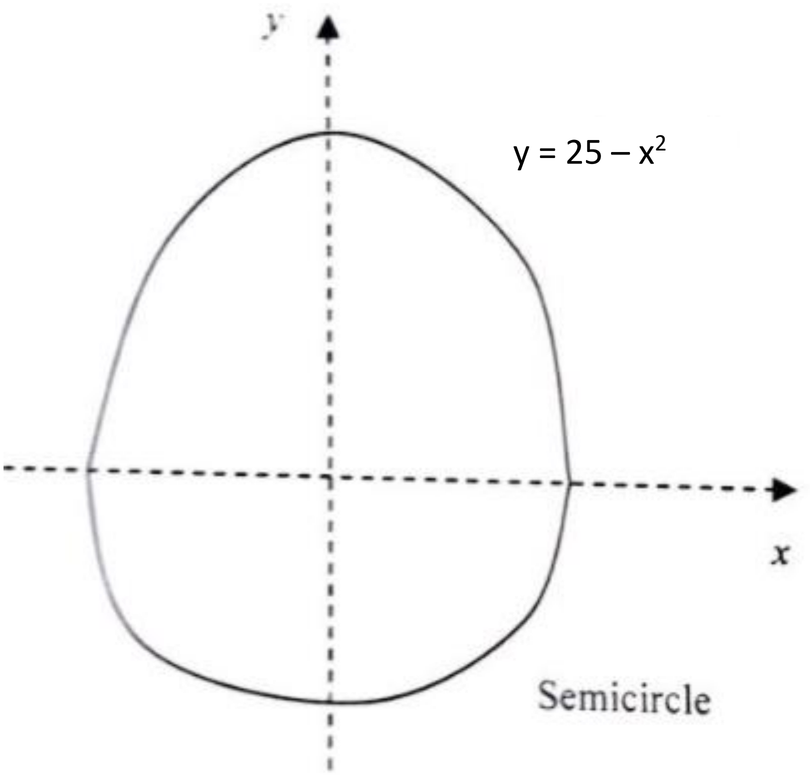
(i) Find the values pf x where the curve crosses the x-axis. [3]

(ii) Find the area between the curve and the positive x-axis. [3]

(iii) Hence find the area of the piece of wood. [1]

Q.4.3.6

The cross section of a baby's toy can be modelled by that part of the curve $y = 25 - x^2$ above the x-axis added on top of a semicircle below the axis, as shown in the diagram below:



(i) Find the area under the curve and above the x-axis. [5]

(ii) Find the area of the cross section of the toy. [2]

5.1 Logarithms – Basic Principles

Q.5.1.1 (*) If $\log a = x$, $\log b = y$ and $\log c = z$, write the following in terms of x, y and z.

(i) $\log\left(\frac{ab}{c^2}\right)$

(ii) $\log\left(\frac{\sqrt{b}}{ac}\right)$

Q.5.1.2 (*) **(a)** Solve the equation

$$\log_4 x = 3$$

(b) If $\log_3 5 = x$ and $\log_3 2 = y$, express the following in terms of x and y only

(i) $\log_3 20$

(ii) $\log_3 7.5$

Q.5.1.3 (*) **(a)** If $y^2 = x^3 z^4$ find $\log z$ in terms of $\log x$ and $\log y$.

(b) If $\log_5 7 = p$ and $\log_5 2 = q$ find

(i) $\log_5 2.5$

(ii) $\log_5 350$

Q.5.1.4

(a) Find the value of x if:

(i) $x = \log_{25} 5$ [1]

(ii) $\log_x 64 = 3$ [1]

(b) If a = $\log_2 6$ and b = $\log_2 5$ express in terms of a and b:

(i) $\log_2 150$ [2]

(ii) $\log_2 0.6$ [3]

Q.5.1.5

(a) Find the value of x if:

(i) $x = \log_2 27$ [1]

(ii) $\log_x 2 = 0.25$ [1]

(b) If $m = \log_{11} 3$ and $n = \log_{11} 4$ express in terms of m and n:

(i) $\log_{11} 36$ [2]

(ii) $\log_{11} 132$ [3]

5.2 Logarithms - log/log Graphs in Context

Q.5.2.1 (*) A small landing craft leaves a main space station orbiting Mars and descends to the surface of the planet. The table below shows the altitude a metres above the Martian surface at which the craft should be when it is travelling at velocity v m/s.

Velocity v (m/s)	**Altitude** a (m)
3500	154 060
2500	129 770
2000	115 810
1500	100 000
1000	81 320

It is believed that a relationship of the form

$$a = kv^n$$

exists, where k and n are constants.

(i) Verify that a relationship of the form $a = kv^n$ exists by drawing a suitable straight line graph. Show clearly the values used, correct to 3 decimal places.

(ii) Hence find the values of k and n.

(iii) Use the formula $a = kv^n$ with your values for k and n to calculate the altitude of the craft when its velocity is 2800 m/s.

The craft is due to reach subsonic speed in the Martian atmosphere at an altitude of 39 000 m.

(iv) Calculate the velocity of the craft when it is at this height. State the assumption which you have made.

Q.5.2.2 (*) An ornithologist measured the masses of birds' eggs and compared these with the masses of the adult birds. The table below shows the body mass b grams and the egg mass e grams for several birds.

Bird	**Bird mass** b (grams)	**Egg mass** e (grams)
Sparrow	30	18.4
Thrush	70	26.9
Tern	130	35.6
Puffin	400	59.0
Heron	1600	110.1
Stork	3400	154.5

It is believed that a relationship of the form

$$e = kb^n$$

exists, where k and n are constants.

(i) Verify that a relationship of the form $e = kb^n$ exists by drawing a suitable straight line graph. Show clearly the values used, correct to 3 decimal places.

(ii) Hence find the values of k and n.

Use the formula $e = kb^n$ with your values for k and n to calculate

(iii) the egg mass for an oystercatcher, whose body mass is 600 grams,

(iv) the body mass of an owl, whose egg mass is 53.4 grams.

Q.5.2.3 The length of a baby was recorded at various stages throughout the first two years of her life. The results are given in the table below.

Age A (months)	**Length** L (cm)
3	57.8
6	65.4
9	70.4
12	74.1
18	79.7
24	84.0

It is believed that a relationship of the form

$$L = kA^n$$

exists, where L is the length in centimetres, A is the age in months, and k and n are constants.

(i) Verify that a relationship of the form $e = kb^n$ exists by drawing a suitable straight line graph. Show clearly the values used, correct to 3 decimal places.

(ii) Hence find the values of k and n.

Use the formula $L = kA^n$ with your values for k and n to calculate

(iii) the length of the baby when she was 15 months old,

(iv) the age at which she was 68 cm long.

(v) Explain briefly why the formula could not hold at the time of birth.

Q.5.2.4 To test the performance of a car, it was driven a distance of 100 km at a uniform speed V km/h. The amount of petrol, P litres, which was used was recorded. This test was repeated for several uniform speeds, and the table below shows the amounts of petrol used on each occasion.

Speed V (km/h)	**Petrol used** P (litres)
50.0	4.37
60.0	5.44
70.0	6.55
85.0	8.27
115.0	11.88

It is believed that a relationship of the form

$$P = kV^n$$

exists, where k and n are constants.

(i) Verify that a relationship of the form $e = kb^n$ exists by drawing a suitable straight line graph. Show clearly the values used, correct to 3 decimal places.

(ii) Hence find the values of k and n.

Use the formula $P = kV^n$ with your values for k and n to calculate

(iii) the speed at which the amount of petrol used would be 5.00 litres,

(iv) the amount of money saved by travelling a distance of 200km at a constant speed of 70.0 km/h rather than at a constant speed of 100 km/h, if the cost of petrol is 120.0 pence per litre.

(iv) Petrol saved = 7 litres, so money saved = £8.40

5.3 Logarithms - Indicial Equations

Q.5.3.1 (*) Solve the equation

$$4^{3x-2} = 5^{1-2x}$$

Q.5.3.2 (*) Solve the equation $16^{1-\frac{1}{2}x} = 7^{2x}$

Q.5.3.3

Solve the equation

$$3^{7-x} = 2^{5x+4}$$

[5]

Q.5.3.4

Solve the equation

$$5^{7x-1} = 3^{x+2}$$

[5]

6.1 Matrices – Basic Operations

Q.6.1.1 (*) Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} -2 & 7 \\ 4 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} -1 & 2 \\ 5 & -3 \end{bmatrix}$$

Calculate

(i) 2**A** + 3**B**

(ii) **C** - 2**A**

Q.6.1.2 (*) Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -4 \\ 6 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} 2 & -3 \end{bmatrix}$$

Calculate

(i) $\mathbf{A}^2$

(ii) **AB**

(iii) **BC**

Q.6.1.3

Simplify $\begin{bmatrix}7\\5\end{bmatrix}$ -2 $\begin{bmatrix}4\\-1\end{bmatrix}$ [1]

Q.6.1.4

Simplify $\begin{bmatrix}3\\4\end{bmatrix}$ +3 $\begin{bmatrix}-5\\2\end{bmatrix}$ [1]

Q.6.1.5 For each matrix product below tick whether the product is possible or not.

If the product is possible, write its value in the space to the right. [10]

(i) $\begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}3 & 0\\1 & 5\end{bmatrix}$ Possible (Yes/No)______

(ii) $\begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}1 & 5\end{bmatrix}$ Possible (Yes/No)____

(iii) $\begin{bmatrix}1 & 5\end{bmatrix}\begin{bmatrix}3\\-1\end{bmatrix}$ Possible (Yes/No)______

(iv) $\begin{bmatrix}3\\-1\end{bmatrix}\begin{bmatrix}1 & 5\end{bmatrix}$ Possible (Yes/No)______

Q.6.1.6 (a)

Evaluate the matrix products:

(i) $\begin{bmatrix}3 & -4\end{bmatrix}\begin{bmatrix}5 & 2\\-8 & 1\end{bmatrix}$ [1]

(ii) $\begin{bmatrix}6\\7\end{bmatrix}\begin{bmatrix}-1 & 2\end{bmatrix}$ [1]

(iii) $\begin{bmatrix}-1 & 5\end{bmatrix}\begin{bmatrix}9\\4\end{bmatrix}$ [1]

6.2 Matrices - Solve Matrix Equations

Q.6.2.1 (*) Two matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ -2 & 3 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}.$$

Solve the equation

$$\mathbf{AX} = \mathbf{B}^2$$

Q.6.2.2 (*) Matrices **A**, **B** and **C** are define by

$$\mathbf{A} = \begin{bmatrix} -2 & 4 \\ 3 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -5 \\ 2 \end{bmatrix} \text{ and } \mathbf{C} = \begin{bmatrix} -3 & -4 \end{bmatrix}$$

Solve the equations

(i) $\mathbf{AX} = \mathbf{B}$

(ii) $2\mathbf{X} = \mathbf{BC}$

Q.6.2.3 Solve for $\begin{bmatrix} x \\ y \end{bmatrix}$ the equation $\begin{bmatrix} 5 \\ -3 \end{bmatrix} - \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ [1]

Q.6.2.4 Find x and y if: $2\begin{bmatrix} x \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -7 \end{bmatrix}$ [1]

Q.6.2.5

Matrices **A** and **B** are defined by:

$\mathbf{A} = \begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

(i) Find 3**A** [1]

(ii) Find the inverse matrix $\mathbf{A}^{-1}$ [2]

(iii) Hence, solve the matrix equation **AX** = **B** [3]

Q.6.2.6

Matrices **A, B** and **C** are defined by:

$\mathbf{A} = \begin{bmatrix} 2 & 5 \\ 4 & 12 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 1 & 4 \\ -3 & 7 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 2 & 5 \\ 3 & 0 \end{bmatrix}$

(i) Find **B** + 2**C** [1]

(ii) Find the inverse matrix $\mathbf{A}^{-1}$ [2]

(iii) Hence, solve the matrix equation: $\mathbf{AX} - \mathbf{B} = 2\mathbf{C}$ [3]

Q.6.2.7

Matrices **P, Q** and **R** are defined by:

$\mathbf{P} = \begin{bmatrix} 3 & 7 \\ 1 & 5 \end{bmatrix}$ $\mathbf{Q} = \begin{bmatrix} -1 & 3 \\ 6 & -1 \end{bmatrix}$ $\mathbf{R} = \begin{bmatrix} 4 & 0 \\ 5 & 2 \end{bmatrix}$

Solve the matrix equation $\mathbf{PX} + \mathbf{Q} = \mathbf{R}$

6.3 Matrices - Solve 2 × 2 Simultaneous Equations

Q.6.3.1 (*) A matrix **A** is defined by

$$\mathbf{A} = \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}.$$

(i) Find the inverse of the matrix **A**.

(ii) **Hence**, using matrices, solve the simultaneous equations

$$4x - 5y = -54$$
$$2x + 3y = 17$$

Q.6.3.2 A matrix **A** is defined by

$$\mathbf{A} = \begin{bmatrix} -6 & 4 \\ -2 & 5 \end{bmatrix}.$$

(i) Find the inverse of the matrix **A**.

(ii) **Hence**, using matrices, solve the simultaneous equations

$$-6x + 4y = 28$$
$$-2x + 5y = 24$$

Q.6.3.3

A matrix is defined by:

$$\mathbf{A} = \begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}$$

(i) Find the inverse matrix $\mathbf{A}^{-1}$ [2]

(II) Hence, solve, using matrices, the simultaneous equations:

$$3x - 2y = 5$$
$$4x - 3y = 7$$

[4]

Q.6.3.4

A matrix is defined by:

$$\mathbf{A} = \begin{bmatrix} 7 & -3 \\ 8 & 4 \end{bmatrix}$$

(i) Find the inverse matrix $\mathbf{A}^{-1}$ [2]

(II) Hence, solve, using matrices, the simultaneous equations:

$$7x - 3y = 4$$
$$8x + 4y = -1$$

[4]

Q6.3.5

A matrix is defined by:

$$\mathbf{A} = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$$

(i) Find the inverse matrix $\mathbf{A}^{-1}$ [2]

(II) Hence, solve, using matrices, the simultaneous equations:

$$4x + 3y = 1$$
$$3x + 2y = -1$$

[4]

Q.6.3.6

A matrix is defined by:

$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 4 & -2 \end{bmatrix}$

(i) Find the inverse matrix $\mathbf{A}^{-1}$ [2]

(II) Hence, solve, using matrices, the simultaneous equations:

$$3x + y = -1$$

$$4x - 2y = 9$$

[4]

Q.6.3.7

A matrix is defined by:

$\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

(i) Find the inverse matrix $\mathbf{A}^{-1}$ [2]

(II) Hence, solve, using matrices, the simultaneous equations:

$$2x - y = 6$$

$$4x + 3y = -5$$

[4]

Answers 1.1 Algebra - Algebraic Fractions

A.1.1.1 (i) $\frac{5}{x^2-x} + \frac{2}{x^2-4x+3}$

$= \frac{5}{x(x-1)} + \frac{2}{(x-1)(x-3)}$

$= \frac{5(x-3) + 2x}{x(x-1)(x-3)}$

$= \frac{7x-15}{x(x-1)(x-3)}$

(ii) $\frac{x^2-4}{x^2-x-6} \div \frac{6}{3x-9}$

$= \frac{x^2-4}{x^2-x-6} \times \frac{3x-9}{6}$

$= \frac{(x+2)(x-2)}{(x-3)(x+2)} \times \frac{3(x-3)}{6}$

$= \frac{(x-2)}{(x-3)} \times \frac{(x-3)}{2}$

$= \frac{(x-2)}{2}$

A.1.1.2 (ii) $x = \frac{15}{4}$ or $x = 6$

A.1.1.3 (i) $\frac{2x-1}{x-2} - \frac{x-3}{6-x} = \frac{(2x-1)(6-x)-(x-2)(x-3)}{(x-2)(6-x)}$

$= \frac{(-2x^2+13x-6)-(x^2-5x+6)}{(-x^2+8x-12)}$

$= \frac{(-3x^2+18x-12)}{(-x^2+8x-12)}$

$= \frac{(3x^2-18x+12)}{(x^2-8x+12)}$

$= \frac{3(x^2-6x+4)}{(x^2-8x+12)}$

(ii) $\frac{2x-1}{x-2} - \frac{x-3}{6-x} = 3$

From **(i)**

$\frac{3(x^2-6x+4)}{(x^2-8x+12)} = 3$

$3(x^2 - 6x + 4) = 3(x^2 - 8x + 12)$

$3x^2 - 18x + 12 = 3x^2 - 24x + 36$

$6x = 24$

$x = 4$

A.1.1.4: (a) $\frac{3x^2 + 10x + 17}{(x+5)(2x+3)}$ (b) $\frac{2(x+1)}{3(x-4)}$

A.1.1.5   (a) $\frac{(x-1)^2}{(x-3)^2}$ (b) 1

A.1.1.6: $\frac{(3x-1)(x+1)}{(x+2)(2x+3)}$

A.1.1.7 $\frac{1}{(x+3)(x-11)}$

Answers 1.2 Algebra - Algebraic Manipulation

A.1.2.1 $(3x + 2)(2x - 1)(x + 1)$

$= (3x + 2)(2x^2 + 2x - x - 1)$

$= (3x + 2)(2x^2 + x - 1)$

$= 3x(2x^2 + x - 1) + 2(2x^2 + x - 1)$

$= 6x^3 + 3x^2 - 3x + 4x^2 + 2x - 2$

$= 6x^3 + 7x^2 - x - 2$

A.1.2.2 $x^3 - 3x^2 - 6x + 8$

A.1.2.3 $x^3 - 11x^2 + 23x + 35$

A.1.2.4 $6x^3 - 19x^2 + 2x + 3$

A.1.2.5 $2x^3 + 9x^2 - 6x - 5$

A.1.2.6 $9x^3 + 9x^2 - x - 1$

Answers 1.3 Algebra - Completing the square

A.1.3.1 $x^2 - 8x - 7 = 0$

$(x^2 - 8x + 16) - 16 - 7 = 0$

$(x - 4)^2 - 23 = 0$

$(x - 4)^2 = 23$

$x - 4 = \pm\sqrt{23}$

$x = 4 \pm\sqrt{23}$

A.1.3.2 **(i)** $y = x^2 - 7x + 3$

$= (x^2 - 7x + \frac{49}{4}) - \frac{49}{4} + 3$

$= (x - \frac{7}{2})^2 - \frac{37}{4}$

(ii) **(a)** Minimum value occurs when $(x - \frac{7}{2})^2 = 0$

$y = 0 - \frac{37}{4}$

Minimum value of the curve is $-\frac{37}{4}$

(b) Minimum value occurs when $(x - \frac{7}{2})^2 = 0$

Minimum value occurs when $x = \frac{7}{2}$

A.1.3.3 $x = 1$ or $x = -7$

A.1.3.4 $x = 3 \pm \sqrt{12}$

A.1.3.5 $(x - 6)^2 - 29$

A.1.3.6 $x = -\frac{1}{2} \pm \sqrt{\frac{17}{2}}$

Answers 1.4 Algebra - Simultaneous equations

A.1.4.1 (i) From the information given

$$20x + 30y + 5z = 100$$

Hence, by dividing by 5

$$4x + 6y + z = 20$$

(ii) From the information given

$$30x + 36y + 24z = 174$$

Hence, by dividing by 6

$$5x + 6y + 4z = 29$$

(ii) Price of Economy $= x$
Price of Standard $= \frac{1}{2}y$
Price of Deluxe $= z - 0.5$

From the information given

$$5x + 40\left(\frac{1}{2}y\right) + 30(z - 0.5) = 111$$

$$5x + 20y + 30z - 15 = 111$$

$$5x + 20y + 30z = 126$$

(iii)

$$4x + 6y + z = 20 \quad [1]$$
$$5x + 6y + 4z = 29 \quad [2]$$
$$5x + 20y + 30z = 126 \quad [3]$$

Eliminate x from equations [1] and [2]

$[1] \times 5 \rightarrow \quad 20x + 30y + 5z = 100$
$[2] \times 4 \rightarrow \quad 20x + 24y + 16z = 116$

By subtraction, $\quad 6y - 11z = -16 \quad [4]$

Eliminate x from equations [2] and [3]

Subtract equation [2] from equation [3]

$$14y + 26z = 97 \quad [5]$$

Eliminate y from equations [4] and [5]

$[4] \times 14 \rightarrow \quad 84y - 154z = -224$
$[5] \times 6 \rightarrow \quad 84y + 156z = 582$

By addition, $\quad 310z = 806$

$$z = 2.6$$

Substitute $z = 2.6$ in equation [4]

$$6y - 11(2.6) = -16$$

$$6y - 28.6 = -16$$

$$6y = 12.6$$

$$y = 2.1$$

Substitute $y = 2.1$ and $z = 2.6$ in equation [1]

$$4x + 6(2.1) + 2.6 = 20$$

$$4x + 12.6 + 2.6 = 20$$

$$4x = 4.8$$

$$x = 1.2$$

Solution is $x = 1.20$, $y = 2.10$ and $z = 2.60$

(v) In the sale

price of Economy = 1.20
price of Standard = $\frac{1}{2}$(2.10) = 1.05
price of Deluxe = 2.60 - 0.50 = 2.10

Total sales = (16 × 1.20) + (80 × 1.05) + (32 × 2.10)

= £170.40

A.1.4.2 **(i)** $5x + 3y + 2z = 74$

(ii) $x - y - z = 1$

(iii) $(4x + 7) + (2y + 4) + (z + 2z) = 72 \rightarrow 4x + 2y + 3z = 61$

(iv) $x = 10$, $y = 6$, $z = 3$

assault - 10 years, arson - 6 years, robbery - 3 years

A.1.4.3 $x = 2,\ y = 6,\ z = 5.5$

A.1.4.4 $x = 3,\ y = 4.1,\ z = 5.2$

Answers 1.5 Algebra - Quadratic Inequalities

A.1.5.1 $x^2 + 7x - 8 < 0$

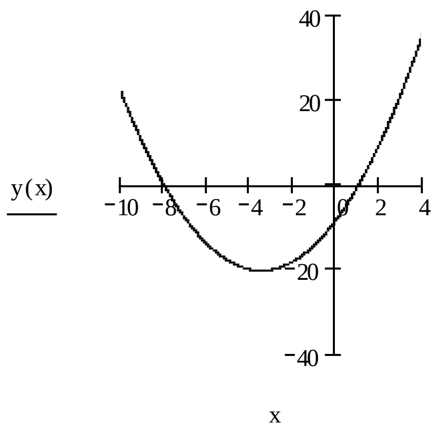
Find the roots of the equation

$x^2 + 7x - 8 = 0$

$(x + 8)(x - 1) = 0$

$x = -8$ or $x = 1$

Sketch the graph of $y = x^2 + 7x - 8$



$x^2 + 7x - 8 < 0$ in the shaded region shown

$x^2 + 7x - 8 < 0$ for $-8 < x < 1$

A.1.5.2 $x^2 - 3x \geq 10$

$x^2 - 3x - 10 \geq 0$

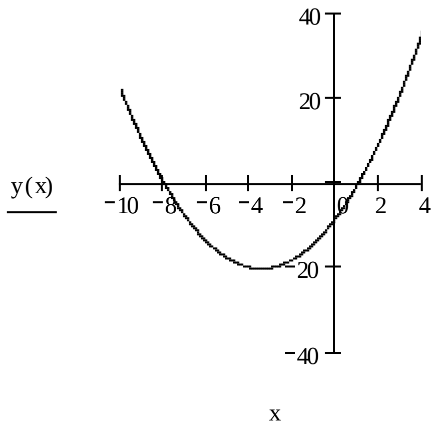
Find the roots of the equation

$x^2 - 3x - 10 = 0$

$(x + 2)(x - 5) = 0$

$x = -2$ or $x = 5$

Sketch the graph of $y = x^2 - 3x - 10$



$x^2 - 3x - 10 \geq 0$ in the shaded regions shown

$x^2 - 3x - 10 \geq 0$, i.e. $x^2 - 3x \geq 10$ for $x \leq -2$ or $x \geq 5$

A.1.5.3 $x \leq -3$ or $x - \geq \frac{1}{2}$

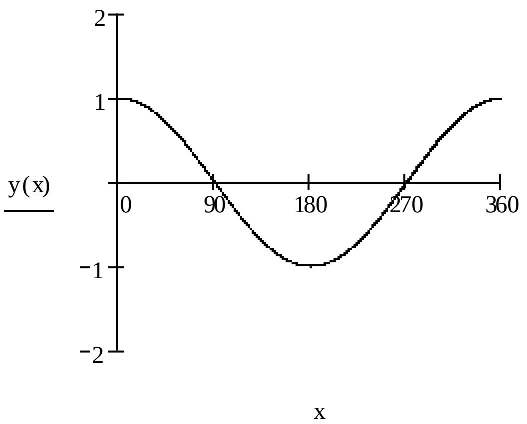
A.1.5.4 $x < -\frac{1}{4}$ or $x > 3$

A.1.5.5 $1 < x < 6$

A.1.5.6 $-\frac{3}{4} \leq x \leq \frac{3}{4}$

Answers 2.1 Trigonometry

A.2.1.1 (i)



(ii) $\cos x = -0.65$

$x = 130.542°$ or $229.458°$

$x = 130.54°$ or $229.46°$ (to 2 decimal places)

(iii) $\cos(2\theta - 10°) = -0.65$

From **(i)**

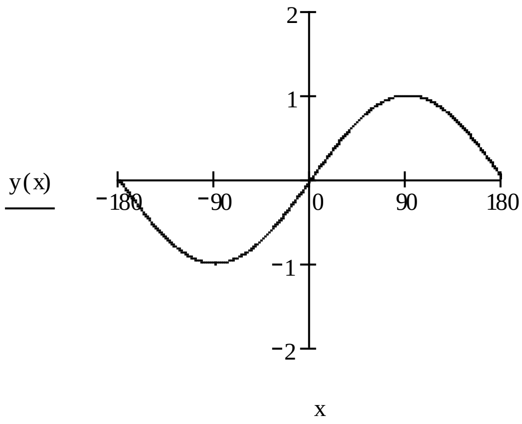
$2\theta - 10° = 130.542°$ or $229.458°$

$2\theta = 140.542°$ or $239.458°$

$\theta = 70.271°$ or $119.729°$

$\theta = 70.27°$ or $119.73°$ (to 2 decimal places)

A.2.1.2 **(i)** $y = \sin x$



(ii) $\sin x = -0.45$

$x = -26.744°$ or $-153.256°$

$x = -26.74°$ or $-153.26°$ (to 2 decimal places)

(iii) $\sin(3\theta + 10°) = -0.45$

From **(i)**

$3\theta + 10° = -26.744°$ or $-153.256°$

$3\theta = -36.744°$ or $-163.256°$

$\theta = -12.248°$ or $-54.419°$

$\theta = -12.25°$ or $-54.42°$ (to 2 decimal places)

A.2.1.3 **(i)** $x = 101.54°$ or $258.46°$ (to 2 decimal places).

(ii) $\theta = 58.27°$ or $136.73°$ (to 2 decimal places).

A.2.1.4 (i) $\theta = 17.46^o \ or \ \theta = 162.54^o$ (ii) $x = 26.23^o \ or \ x = 63.77^o$

A.2.1.5 (i) $\theta = -30.96^o or \ \theta = -149.04^o$ (ii) $x = -41.93^o \ or \ x = -278.07^o$

Answers 3.1 Differentiation - basic

A.3.21.1 $y = \frac{2}{7}x^{14} - \frac{14}{x^7} = \frac{2}{7}x^{14} - 14x^{-7}$

$\frac{dy}{dx} = \frac{28}{7}x^{13} + 98x^{-8}$

$= 4x^{13} + 98x^{-8}$ or $4x^{13} + \frac{98}{x^8}$

A.3.1.2 $y = 8x^3 - 2 + \frac{4}{x^2}$

$y = 8x^3 - 2 + 4x^{-2}$

$\frac{dy}{dx} = 24x^2 - 8x^{-3}$

$\frac{d^2y}{dx^2} = 48x + 24x^{-4}$ or $48x + \frac{24}{x^4}$

A.3.1.3 : (i) $\frac{dy}{dx} = 24x^5 + \frac{10}{x^3}$ (ii) $\frac{d^2y}{dx^2} = 120x^4 - \frac{30}{x^4}$

A.3.1.4 (i) $\frac{dy}{dx} = 6x + \frac{2}{x^2}$ (ii) $\frac{d^2y}{dx^2} = 6 - \frac{4}{x^3}$

A.3.1.5 (i) $\frac{dy}{dx} = 8x^7 - \frac{35}{x^6}$ (ii) $\frac{d^2y}{dx^2} = 56x^6 - \frac{210}{x^7}$

A.3.1.6 $y = 4x^3 - 12x^2 + 21x + 2$

$\frac{dy}{dx} = 12x^2 - 24x + 21$

$\frac{d^2y}{dx^2} = 24x - 24$

$12x^2 - 24x + 21 = 24x - 24$

$12x^2 - 48x + 45 = 0$

$4x^2 - 16x + 15 = 0$

$(2x - 5)(2x - 3) = 0$

$x = \frac{5}{2}$ or $x = \frac{3}{2}$

Answers 3.2 Differentiation - finding equations of tangents and normals at points on a curve

A.3.2.1 **(i)** $y = 2x + \frac{3}{4x} = 2x + \frac{3}{4}x^{-1}$

$\frac{dy}{dx} = 2 - \frac{3}{4}x^{-2} = 2 - \frac{3}{4x^2}$

At P, $x = 1$, $\frac{dy}{dx} = 2 - \frac{3}{4} = \frac{5}{4}$

So gradient of tangent at P $= \frac{5}{4}$

For equation of tangent, $x = -1$, $y = -2\frac{3}{4}$, $m = \frac{5}{4}$

$y = mx + c$

$-2\frac{3}{4} = \frac{5}{4}(-1) + c$

$-\frac{11}{4} = -\frac{5}{4} + c$

$c = -\frac{11}{4} + \frac{5}{4} = -\frac{6}{4} = -\frac{3}{2}$

Equation of tangent is $y = \frac{5}{4}x - \frac{3}{2}$

(ii) Gradient of normal at P $= \frac{-1}{\text{gradient of tangent at P}} = \frac{-1}{\left(\frac{5}{4}\right)} = -\frac{4}{5}$

Line $15y + 12x = 7$ can be written as

$15y = -12x + 7$

$y = -\frac{12}{15}x + \frac{7}{15}$

$y = -\frac{4}{5}x + \frac{7}{15}$

Gradient of line $= -\frac{4}{5}$ = gradient of normal at P

Hence the line and the normal are parallel.

A.3.2.2 $y = ax^2 - 5x + b$

$\frac{dy}{dx} = 2ax - 5$

At $x = 2$, $\frac{dy}{dx} = 4a - 5$ = gradient of tangent at (2, 6)

Equation of tangent at (2, 6) is $y = 7x - 8$

So gradient of tangent at (2, 6) is 7

Hence $4a - 5 = 7$

$4a = 12$

$a = 3$

So $y = 3x^2 - 5x + b$

Curve passes through (2, 6)

So $6 = 3(2^2) - 5(2) + b$

$6 = 12 - 10 + b$

$b = 6 - 12 + 10$

$b = 4$

A.3.2.3 (i) $y = 7x - \frac{2}{x^2} = 7x - 2x^{-2}$

$\frac{dy}{dx} = 7 + 4x^{-3} = 7 + \frac{4}{x^3}$

At P, $x = -1$, $\frac{dy}{dx} = 7 - 4 = 3$

So gradient of tangent at P $= 3$

For equation of tangent, $x = -1$, $y = -9$, $m = 3$

$y = mx + c$

$-9 = 3(-1) + c$

$c = -9 + 3 = -6$

Equation of tangent is $y = 3x - 6$

(ii) Gradient of normal at P $= \frac{-1}{\text{gradient of tangent at P}} = -\frac{1}{3}$

Equation of normal is $y = -\frac{1}{3}x + c$

At P, $-9 = -\frac{1}{3}(-1) + c$

$c = -9\frac{1}{3}$

Equation of normal is $y = -\frac{1}{3}x - 9\frac{1}{3}$

This meets the line where

$-\frac{1}{3}x - 9\frac{1}{3} = 5 - 2x$

$-x - 28 = 15 - 6x$

$5x = 43$

$x = 8.6$

Normal meets the line at the point (8.6, - 12.2)

A.3.2.4 **(i)** Equation of tangent is $y = 2x + 2$

(ii) Tangent meets the line at the point (1, 4).

A.3.2.5 **(i)** Equation of tangent is $y = 3x - 5$

(ii) Equation of normal is $y = -\frac{1}{3}x + \frac{5}{3}$

(iii) Normal cuts the curve at the point $(\frac{1}{3}, 4\frac{2}{3})$.

A.3.2.6 **(i)** Coordinates are (0.8, 2.04) and (- 0.33, - 0.93)

(ii) Equation of normal is $y = \frac{1}{15}x + 3\frac{1}{15}$

A.3.2.7 (i) $y = 3x - 5$ (ii) $(0, \frac{5}{3})$

A.3.2.8 (i) $y = 8x + 16$ (ii) $y = -\frac{1}{8}x + \frac{1}{4}$ (iii) $(-\frac{126}{65}, \frac{32}{65})$

Answers 3.3 Differentiation - Simple Optimisation Problems

A.3.3.1 (i) Max P = £200,400 after 200 months (ii) $\frac{d^2P}{dt^2} = -10\,, negative\ so\ P\ is\ a\ maximum$

A.3.3.2 (i) Type equation here.= 80,000$t^3 - 270{,}000$ (ii) 196,250 cells when t = 1.5s

(iii) $\frac{d^2C}{dt^2} = 240{,}000t^2 = 540{,}000 > 0\ thus\ a\ minimum\ value$

A.3.3.3 (ii) (4, 3) or (4.8, 1.4) (iii) (4.4, 2.2) and $\frac{d^2A}{dx^2} = 10, positive\ thus\ a\ minimum$

A.3.3.4 (ii) A = 8x + 10y (iii) A max = 921.6cm^2 and $\frac{d^2A}{dx^2} = -12.8 < 0\ thus\ maximum$

Answers 3.4 Differentiation - Elementary Curve Sketching of a Quadratic or Cubic Function

A.3.4.1 **(i)** $y = 21 - 4x - x^2$

Curve crosses the x-axis when $y = 0$

$21 - 4x - x^2 = 0$

$x^2 + 4x - 21 = 0$

$(x + 7)(x - 3) = 0$

$x = -7$ or $x = 3$

Coordinates of points are $(-7, 0)$ and $(3, 0)$

(ii) $\frac{dy}{dx} = -4 - 2x$

For a turning point, $\frac{dy}{dx} = 0$

$-4 - 2x = 0$

$2x = -4$

$x = -2$

When $x = -2$, $y = 21 - 4x - x^2$

$= 21 - 4(-2) - (-2)^2$

$= 21 + 8 - 4$

$= 25$

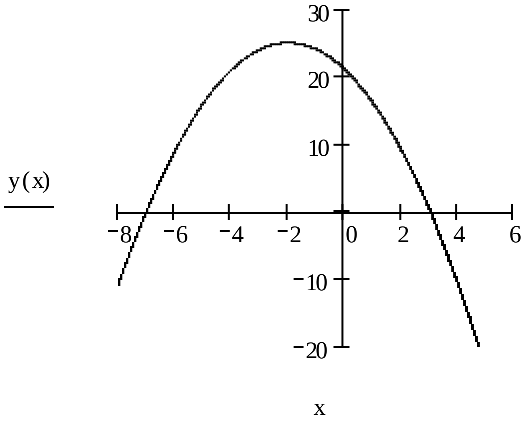
Coordinates of turning point are $(-2, 25)$

(iii) $\frac{d^2y}{dx^2} = -2$

$\frac{d^2y}{dx^2} < 0$

So turning point at $(-2, 25)$ is a maximum point

(iv)



(v) Area $= \int_{-2}^{0} (21 - 4x - x^2)\, dx$

$= [\,21x - 2x^2 - \frac{1}{3}x^3]_{-2}^{0}$

$= [\,0\,] - [\,21(-2) - 2(-2)^2 - \frac{1}{3}(-2)^3\,]$

$= [\,0\,] - [\,-42 - 8 + \frac{8}{3}\,]$

$= -[\,-47\frac{2}{3}\,]$

$= 47\frac{2}{3}$

A.3.4.2 **(i)** $y = 2x(3 - x)(5 + 2x)$

Curve crosses the x-axis when $y = 0$

$y = 0$ when $x = 0$, $x = 3$ or $x = -\frac{5}{2}$

Coordinates of points are $(-\frac{5}{2}, 0)$, $(0, 0)$ and $(3, 0)$

(ii) $y = 2x(3 - x)(5 + 2x)$

$y = 2x(15 + x - 2x^2)$

$y = 30x + 2x^2 - 4x^3$

$\frac{dy}{dx} = 30 + 4x - 12x^2$

For a turning point, $\frac{dy}{dx} = 0$

$12x^2 - 4x - 30 = 0$

$x = \frac{4 \pm \sqrt{16+1440}}{24}$

$x = 1.76$ or -1.42

When $x = 1.76$, $y = 37.19$

When $x = -1.42$, $y = -27.11$

Coordinates of turning points are $(1.76, 37.19)$ and $(-1.42, -27.11)$

(iii) $\frac{d^2y}{dx^2} = 4 - 24x$

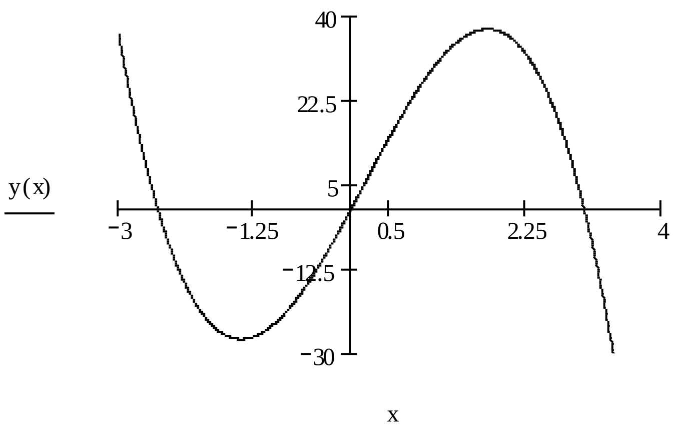
When $x = 1.76$, $\frac{d^2y}{dx^2} = -38.24 < 0$

So turning point at $(1.76, 37.19)$ is a maximum point.

When $x = -1.42$, $\frac{d^2y}{dx^2} = 38.08 > 0$

So turning point at $(-1.42, -27.11)$ is a minimum point.

(iv)

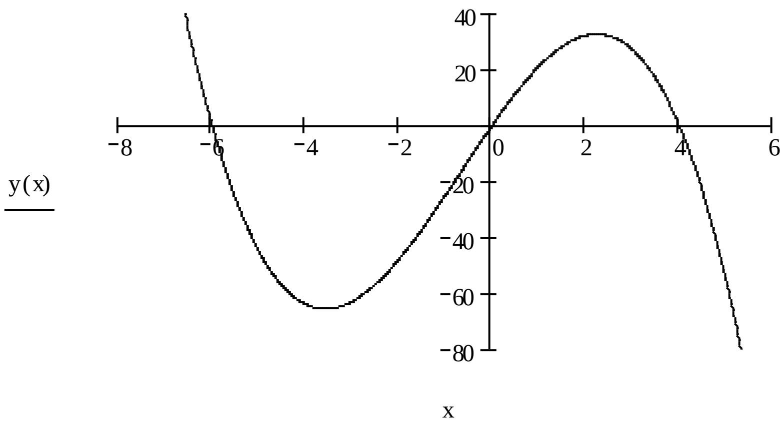


A.3.4.3 **(i)** Crosses the x-axis at the points $(-6, 0)$, $(0, 0)$ and $(4, 0)$.

(ii) Turning points are at $(2.2, 32.5)$ and $(-3.6, -65.7)$ (to 1 decimal place).

(iii) Maximum point at $(2.2, 32.5)$, minimum point at

(iv) $y = 24x - 2x^2 - x^3$



A.3.4.4 (i) (-16,0) (0,0) (5,0) (ii)+(iii) (-8/3, 272.6) MAX and (10, 1300) MIN (v) A = 385.42

A.3.4.5 (i) (-8,0) (0,0) (7,0) (ii)+(iii) (-14/3,161.7) MAX (4,-144) MIN (v) Area = 80/3

.3.4.6 (i) (-10,0) (0,0) (6,0) (ii)+(iii) (-6, 288) MAX (10/3, -118.52) (v) Area = 253.1

A.3.4.7 (i) (-24,0) (-9,0) (0,0) (ii) and (iii) (-18, 972) MAX and (-4, -400) MIN

A.3.4.8 (i) (-24,0) (-15,0) (0,0) (ii) and (iii) (-20,400) MAX and (-6, 972)

Answers 4.1 Integration – Indefinite Integration

A.4.1.1 $\int\left(3x^5+\frac{1}{4x^3}-1\right)\mathrm{d}x$

$= \int\left(3x^5+\frac{1}{4}x^{-3}-1\right)\mathrm{d}x$

$= \frac{3}{6}x^6-\frac{1}{8}x^{-2}-x+c$ (c is a constant)

$= \frac{1}{2}x^6-\frac{1}{8}x^{-2}-x+c$ or $\frac{1}{2}x^6-\frac{1}{8x^2}-x+c$

A.4.1.2 $-\frac{3}{2x^2}-\frac{4}{3}x + c$

A.4.1.3 $\frac{x^4}{4}-\frac{1}{x^2} + c$

A.4.1.4 $6x-\frac{1}{5}x^5 + c$

A.4.1.5 $2x^3 + \frac{5}{28x^4} + c$

A.4.1.6 $-\frac{2}{5x^2} + \frac{7}{5}x^5 + c$

A.4.1.7 $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 3$

$y = \int(4x-3)\mathrm{d}x$

$y = 2x^2 - 3x + c$ (c is a constant)

Substitute $x = 2$, $y = 6$

$6 = 8 - 6 + c$

$c = 4$

$y = 2x^2 - 3x + 4$

Answers 4.2 Integration - Form and Evaluate Definite Integrals

A.4.2.1 $-\frac{4}{3}$

A.4.2.2 87

A.4.2.3 -30

A.4.2.4 $-1019\frac{1}{4}$

Answers 4.3 Integration - Finding the Area Under a Curve

A.4.3.1 $y = 2x^2 - 5x - 3$

The curve cuts the x-axis where

$2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$x = -\frac{1}{2}$ or $x = 3$

Area $= \int_{-½}^{3}(2x^2 - 5x - 3)\mathrm{d}x$

$= [\frac{2}{3}x^3 - \frac{5}{2}x^2 - 3x]_{-½}^{3}$

$= [18 - 22\frac{1}{2} - 9] - [-\frac{1}{12} - \frac{5}{8} + \frac{3}{2}]$

$= [-13\frac{1}{2}] - [\frac{19}{24}]$

$= -14\frac{7}{24}$

A.4.3.2 Area $= \int_{1}^{3}(3x^2 + 4x + 1)\mathrm{d}x$

$= [x^3 + 2x^2 + x]_1^3$

$= [27 + 18 + 3] - [1 + 2 + 1]$

$= [48] - [4]$

$= 44$

A.4.3.3 (i) 36 (ii) 24

A.4.3.4 (i) 36 (ii) 60

A.4.3.5 (i) 0.25 (ii) 0.5

A.4.3.6 (i) $\frac{500}{3}$ (ii) $\frac{500}{3} + \frac{25\pi}{2} = 205.94$

Answers 5.1 Logarithms – Basic Principles

A.5.1.1 (i) $\log\left(\frac{ab}{c^2}\right) = \log(ab) - \log c^2$

$= \log a + \log b - 2\log c$

$= x + y - 2z$

(ii) $\log\left(\frac{\sqrt{b}}{ac}\right) = \log\sqrt{b} - \log(ac)$

$= \log b^{1/2} - (\log a + \log c)$

$= \frac{1}{2}\log b - \log a - \log c$

$= \frac{1}{2}y - x - z$

A.5.1.2 (a) $\log_4 x = 3$

$x = 4^3$

$x = 64$

(b) (i) $\log_3 20 = \log_3(5 \times 2 \times 2)$

$= \log_3 5 + \log_3 2 + \log_3 2$

$= x + y + y$

$= x + 2y$

(ii) $\log_3 7.5 = \log_3\left(\frac{5\times 3}{2}\right)$

$= \log_3 5 + \log_3 3 - \log_3 2$

$= x + 1 - y$

A.5.1.3 **(a)** $y^2 = x^3z^4$

$\log(y^2) = \log(x^3z^4)$

$\log(y^2) = \log(x^3) + \log(z^4)$

$2\log y = 3\log x + 4\log z$

$\log z = \dfrac{2\log y - 3\log x}{4}$

(b) $\log_5 7 = p, \quad \log_5 2 = q$

(i) $\log_5 2.5 = \log_5 \frac{5}{2}$

$\log_5 2.5 = \log_5 5 - \log_5 2$

$\log_5 2.5 = 1 - q$

(ii) $\log_5 350 = \log_5(2 \times 5 \times 5 \times 7)$

$\log_5 350 = \log_5 2 + \log_5 5 + \log_5 5 + \log_5 7$

$\log_5 350 = q + 1 + 1 + p$

$\log_5 350 = p + q + 2$

A.5.1.4 (a)(i) 0.5 (ii) 4 (b)(i) a + 2b (ii) a – b - 1

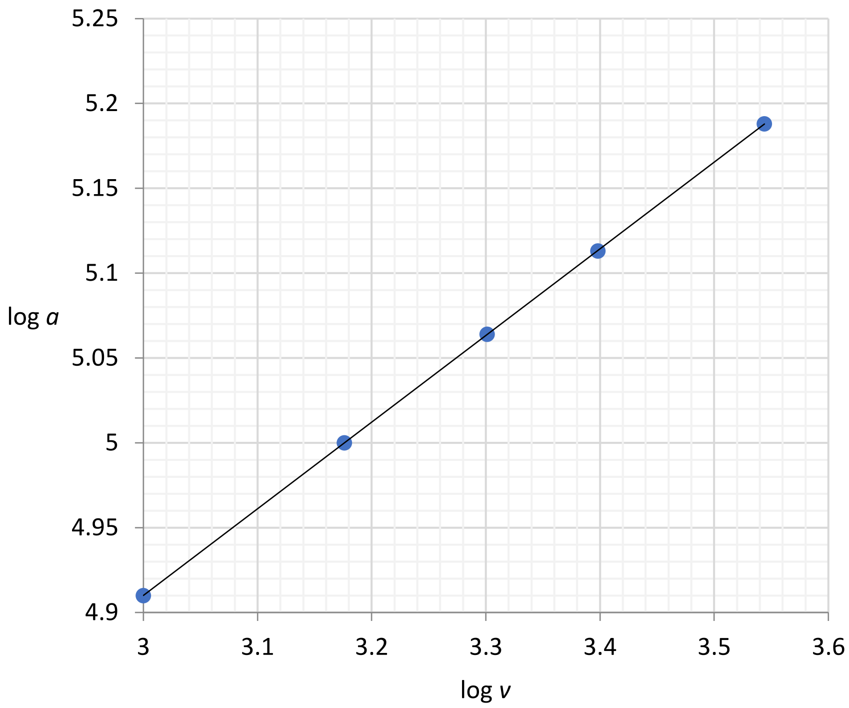
A.5.1.5 (a)(i) 5 (ii) 16 (b)(i) 2m + n (ii) 1 + m + n

Answers 5.2 Logarithms - log/log Graphs in Context

A.5.2.1 **(i)** If $a = kv^n$ then $\log a = n\log v + \log k$

So a straight-line graph of $\log a$ against $\log v$ verifies the relationship.

Velocity v (m/s)	**Altitude** a (m)	$\log v$	Log a
3500	154 060	3.544	5.188
2500	129 770	3.398	5.133
2000	115 810	3.301	5.064
1500	100 000	3.176	5.000
1000	81 320	3.000	4.910



n = gradient of line $= \frac{5.188-4.910}{3.544-3.000} = 0.51$

Using the first given data values,

$$154\ 060 = k\ 3500^{0.51}$$

$$k = 2400$$

The relationship is $a = 2400\ v^{0.51}$

(ii) $a = 2400 \times 2800^{0.51} = 137\ 487$ m

(iii) $39\ 000 = 2400\ v^{0.51}$

$$v^{0.51} = \frac{39\ 000}{2400}$$

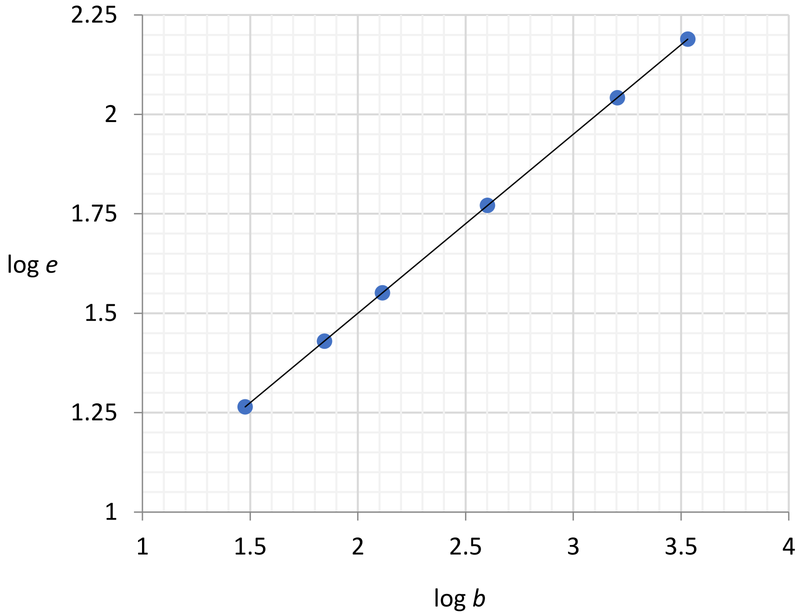
$$v = \left(\frac{39\ 000}{2400}\right)^{\frac{1}{0.51}} = 237 \text{ m/s}$$

The assumption made is that the formula holds for altitudes outside the given range.

A.5.2.2 **(i)** If $e = kb^n$ then $\log e = n \log b + \log k$

So a straight-line graph of $\log e$ against $\log b$ verifies the relationship.

Bird	**Bird mass** b (grams)	**Egg mass** e (grams)	$\log b$	$\log e$
Sparrow	30	18.4	1.477	1.265
Thrush	70	26.9	1.845	1.430
Tern	130	35.6	2.114	1.551
Puffin	400	59.0	2.602	1.771
Heron	1600	110.1	3.204	2.042
Stork	3400	154.5	5.531	2.189



n = gradient of line $= \frac{2.189-1.265}{3.531-1.477} = 0.45$

Using the last given data values,

$$154.5 = k\, 3400^{0.45}$$

$$k = 3.98$$

The relationship is $e = 3.98\, b^{0.45}$

(ii) $a = 3.98 \times 600^{0.51} = 70.8$ grams

(iii) $53.4 = 3.98\, b^{0.45}$

$$b^{0.45} = \frac{53.4}{3.98}$$

$$v = \left(\frac{53.4}{3.98}\right)^{\frac{1}{0.45}} = 321 \text{ grams}$$

A.5.2.3 **(ii)** $n = 0.18$ and $k = 47.4$

The relationship is $L = 47.4\,A^{0.18}$

(iii) When $A = 15$, $L = 77.2$ cm

(iv) When $L = 68$, $A = 7.43$ months

(v) At birth $A = 0$. This would give $L = 0$, which is impossible.

A.5.2.4 **(ii)** $n = 1.20$ and $k = 0.04$

The relationship is $P = 0.04\,V^{1.20}$

(iii) When $P = 5$, $V = 55.9$ km/h

(iv) Petrol saved = 7 litres, so money saved = £8.40

Answers 5.3 Logarithms - Indicial Equations

A.5.3.1 $4^{3x-2} = 5^{1-2x}$

Take logs of both sides

$\log 4^{3x-2} = \log 5^{1-2x}$

$(3x - 2)\log 4 = (1 - 2x)\log 5$

$3x\log 4 - 2\log 4 = \log 5 - 2x\log 5$

$3x\log 4 + 2x\log 5 = \log 5 + 2\log 4$

$x(3\log 4 + 2\log 5) = \log 5 + 2\log 4$

$$x = \frac{\log 5 + 2\log 4}{3\log 4 + 2\log 5}$$

$x = 0.59$ (to 2 decimal places)

A.5.3.2 $16^{1 - \frac{1}{2}x} = 7^{2x}$

Take logs of both sides

$\log(16^{1 - \frac{1}{2}x}) = \log(7^{2x})$

$(1 - \frac{1}{2}x)\log 16 = 2x\log 7$

$\log 16 - \frac{1}{2}x\log 16 = 2x\log 7$

$2x\log 7 + \frac{1}{2}x\log 16 = \log 16$

$x(2\log 7 + 0.5\log 16) = \log 16$

$$x = \frac{\log 16}{2\log 7 + 0.5\log 16}$$

$x = 0.53$ (to 2 decimal places)

A.5.3.3 $x = \frac{7\,log\,3 - 4\,log\,2}{5\,log\,2 + log\,3} = 1.08$

A.5.3.4: $x = \frac{2\,log\,3 + log\,5}{7\,log\,5 - log\,3} = 0.37$

Answers 6.1 Matrices – Basic Operations

A.6.1.1 (i) $2\mathbf{A} + 3\mathbf{B} = 2\begin{bmatrix}-2 & 7\\ 4 & 3\end{bmatrix} + 3\begin{bmatrix}5 & -2\\ -3 & 4\end{bmatrix}$

$= \begin{bmatrix}-4 & 14\\ 8 & 6\end{bmatrix} + \begin{bmatrix}15 & -6\\ -9 & 12\end{bmatrix}$

$= \begin{bmatrix}11 & 8\\ -1 & 18\end{bmatrix}$

(ii) $\mathbf{C} - 2\mathbf{A} = \begin{bmatrix}-1 & 2\\ 5 & -3\end{bmatrix} - 2\begin{bmatrix}-2 & 7\\ 4 & 3\end{bmatrix}$

$= \begin{bmatrix}-1 & 2\\ 5 & -3\end{bmatrix} - \begin{bmatrix}-4 & 14\\ 8 & 6\end{bmatrix}$

$= \begin{bmatrix}3 & -12\\ -3 & -9\end{bmatrix}$

A.6.1.2 **(i)** $\mathbf{A}^2 = \begin{bmatrix}-2 & 4\\ 3 & 5\end{bmatrix}\begin{bmatrix}-2 & 4\\ 3 & 5\end{bmatrix}$

$= \begin{bmatrix}4 + 12 & -8 + 20\\ -6 + 15 & 12 + 25\end{bmatrix}$

$= \begin{bmatrix}16 & 12\\ 9 & 37\end{bmatrix}$

(ii) $\mathbf{AB} = \begin{bmatrix}-2 & 4\\ 3 & 5\end{bmatrix}\begin{bmatrix}-4\\ 6\end{bmatrix}$

$= \begin{bmatrix}8+24\\ -12+30\end{bmatrix}$

$= \begin{bmatrix}32\\ 18\end{bmatrix}$

(iii) $\mathbf{BC} = \begin{bmatrix}-4\\ 6\end{bmatrix}\begin{bmatrix}2 & -3\end{bmatrix}$

$= \begin{bmatrix}-8 & 12\\ 12 & -18\end{bmatrix}$

A.6.1.3 $\begin{bmatrix}-1\\ 7\end{bmatrix}$

A.6.1.4 $\begin{bmatrix}-12\\ 10\end{bmatrix}$

A.6.1.5 (i) Yes $\begin{bmatrix}7 & 5\\ -1 & 10\end{bmatrix}$ (ii) No (iii) Yes $\begin{bmatrix}-2\end{bmatrix}$ (iv) Yes $\begin{bmatrix}3 & 15\\ -1 & 10\end{bmatrix}$

A.6.1.6 (i) $\begin{bmatrix}47 & 2\end{bmatrix}$ (ii) $\begin{bmatrix}-6 & 12\\ -7 & 14\end{bmatrix}$ (iii) $\begin{bmatrix}11\end{bmatrix}$

Answers 6.2 Matrices - Solve Matrix Equations

A.6.2.1 $\mathbf{B}^2 = \mathbf{B}\times\mathbf{B} = \begin{bmatrix}4 & -3\\ -2 & 1\end{bmatrix}\begin{bmatrix}4 & -3\\ -2 & 1\end{bmatrix}$

$\begin{bmatrix}16+6 & -12-3\\ -8-2 & 6+1\end{bmatrix} = \begin{bmatrix}22 & -15\\ -10 & 7\end{bmatrix}$

$\mathbf{AX} = \mathbf{B}^2$

$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}^2$

$\mathbf{A} = \begin{bmatrix}-5 & 2\\ -2 & 3\end{bmatrix}$

$\mathbf{A}^{-1} = \frac{1}{-15-(-4)}\begin{bmatrix}3 & -2\\ 2 & -5\end{bmatrix}$

$= -\frac{1}{11}\begin{bmatrix}3 & -2\\ 2 & -5\end{bmatrix}$

$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}^2$

$= -\frac{1}{11}\begin{bmatrix}3 & -2\\ 2 & -5\end{bmatrix}\begin{bmatrix}22 & -15\\ -10 & 7\end{bmatrix}$

$= -\frac{1}{11}\begin{bmatrix}66+20 & -45-14\\ 44+50 & -30-35\end{bmatrix} = -\frac{1}{11}\begin{bmatrix}86 & -59\\ 94 & -65\end{bmatrix}$

A.6.2.2 **(i)** **AX** = **B**

X = $\mathbf{A}^{-1}\mathbf{B}$

A = $\begin{bmatrix} -2 & 4 \\ 3 & 1 \end{bmatrix}$

$\mathbf{A}^{-1} = \frac{1}{-2-12}\begin{bmatrix} 1 & -4 \\ -3 & -2 \end{bmatrix}$

$\mathbf{A}^{-1} = \frac{-1}{14}\begin{bmatrix} 1 & -4 \\ -3 & -2 \end{bmatrix}$

X = $\mathbf{A}^{-1}\mathbf{B}$

$\mathbf{X} = \frac{-1}{14}\begin{bmatrix} 1 & -4 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} -5 \\ 2 \end{bmatrix}$

$\mathbf{X} = \frac{-1}{14}\begin{bmatrix} -5-8 \\ 15-4 \end{bmatrix}$

$\mathbf{X} = \frac{-1}{14}\begin{bmatrix} -13 \\ 11 \end{bmatrix}$

$\mathbf{X} = \begin{bmatrix} 13/14 \\ -11/14 \end{bmatrix}$

(ii) $\mathbf{BC} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}\begin{bmatrix} -3 & -4 \end{bmatrix}$

$\mathbf{BC} = \begin{bmatrix} 15 & 20 \\ -6 & -8 \end{bmatrix}$

2**X** = **BC**

$\mathbf{X} = \frac{1}{2}\begin{bmatrix} 15 & 20 \\ -6 & -8 \end{bmatrix}$

$\mathbf{X} = \begin{bmatrix} 7\frac{1}{2} & 10 \\ -3 & -4 \end{bmatrix}$

A.6.2.3 $\begin{bmatrix} 7 \\ -4 \end{bmatrix}$

A.6.2.4 $x = 4, y = -5$

.6.2.5 (i) $\begin{bmatrix} 6 & -3 \\ -9 & 15 \end{bmatrix}$ (ii) $\frac{1}{7}\begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}$ (iii) $\frac{1}{7}\begin{bmatrix} 8 & 2 \\ 9 & 4 \end{bmatrix}$

A.6.2.6 (i) $\begin{bmatrix} 5 & 14 \\ 3 & 7 \end{bmatrix}$ (ii) $\frac{1}{4}\begin{bmatrix} 12 & -5 \\ -4 & 2 \end{bmatrix}$ (iii) $\frac{1}{4}\begin{bmatrix} 45 & 133 \\ -14 & -42 \end{bmatrix}$

.6.2.7 $\frac{1}{4}\begin{bmatrix} 9 & -18 \\ -3 & 6 \end{bmatrix}$

Answers 6.3 Matrices - Solve 2 × 2 Simultaneous Equations

A.6.3.1 (i) $\mathbf{A} = \begin{bmatrix} 4 & -5 \\ 2 & 3 \end{bmatrix}$

$\mathbf{A}^{-1} = \frac{1}{12-(-10)}\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$

$= \frac{1}{22}\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$

(ii) The equations can be written in the matrix form

$$\begin{bmatrix} 4 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -54 \\ 17 \end{bmatrix}$$

i.e. $\mathbf{A}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -54 \\ 17 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{A}^{-1}\begin{bmatrix} -54 \\ 17 \end{bmatrix}$$

$$= \frac{1}{22}\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -54 \\ 17 \end{bmatrix}$$

$$= \frac{1}{22}\begin{bmatrix} -162 + 85 \\ 108 + 68 \end{bmatrix}$$

$$= \frac{1}{22}\begin{bmatrix} -77 \\ 176 \end{bmatrix}$$

$$= \begin{bmatrix} -3.5 \\ 8 \end{bmatrix}$$

Solution of equations is $x = 3.5$, $y = 8$

A.6.3.2 (i) $\mathbf{A}^{-1} = -\frac{1}{22}\begin{bmatrix} 5 & -4 \\ 2 & -6 \end{bmatrix}$

(ii) $x = -2$, $y = 4$

A.6.3.3 (i) $\begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$ (ii) x = 1, y = -1

A.6.3.4 (i) $\frac{1}{52}\begin{bmatrix} 4 & 3 \\ -8 & 7 \end{bmatrix}$ (ii) $x = \frac{1}{4}$, $y = -\frac{3}{4}$

A.6.3.5 (i) $\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$ (ii) $x = -5, y = 7$

A.6.3.6 (i) $\frac{1}{10}\begin{bmatrix} -2 & 1 \\ 4 & 3 \end{bmatrix}$ (ii) $x = 1.1, y = 2.3$

A.6.3.7 (i) $\frac{1}{10}\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$ (ii) $x = 1{,}3, \ y = -3.4$

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