Rewarding Learning

Summer 2021
Extra Assessment Resources
GCE Further Mathematics A21

Version 1.0

CCEA Extra Assessment Resource GCE Further Maths A21

Show clearly the development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1

Find, in terms of n, the sum of the series

$$1^2 - 2^2 + 3^2 - 4^2 + \;\ldots\ldots\ldots\; + (2n-1)^2 - (2n)^2$$ [4]

2

The sequence $u_1,\ u_2,\ u_3,\ \ldots\ldots\ldots$ is defined by

$$u_1 = 3 \qquad u_{n+1} = 4u_n + 1$$

Using mathematical induction, prove that

$$u_n = \frac{1}{3}\left[5(2^{2n-1}) - 1\right] \qquad \text{for all } n \geqslant 1$$ [5]

3

(b) (i) Solve the equation

$$z^6 = 64$$

giving your answers in $re^{i\theta}$ form. [5]

(ii) Sketch on an Argand diagram the hexagon whose vertices represent the solutions to

$$z^6 = 64$$ [2]

(iii) State the length of the sides of this regular hexagon. [1]

4

(i) Obtain the general solution of the differential equation

$$\tan x \frac{dy}{dx} + y = \sin x \tan x$$ [6]

(ii) Hence find the particular solution, given that $y = \dfrac{1}{2\sqrt{2}}$ when $x = \dfrac{\pi}{4}$ [2]

5

(i) Given that

$$f(x) = e^{-mx} - (1 + 2x)^{-n}$$

find the Maclaurin expansion for $f(x)$ up to and including the term in x^2 [6]

(ii) Given that the first non-zero term in this expansion is $-4x^2$, find the values of m and n. [3]

6

(i) Given that

$$(\cos\theta + i\sin\theta)^n \equiv \cos n\theta + i\sin n\theta$$

when n is a positive integer, deduce that the statement is also true when n is a negative integer. [4]

(ii) Using **(i)**, show that if $Z = \cos\theta + i\sin\theta$, then

$$Z^n + Z^{-n} = 2\cos n\theta$$ [2]

(iii) By considering $(Z + Z^{-1})^4$, show that

$$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$$ [3]

7

Consider the function

$$y = \sin^{-1}\frac{x}{\sqrt{1-x}}$$

(i) Show that

$$\frac{dy}{dx} = \frac{1 - \frac{1}{2}x}{(1-x)\sqrt{1-x-x^2}}$$ [6]

(ii) Hence find the exact equation of the tangent to the curve of the function at the point where $x = -1$ [3]

8

(i) Show that

$$\sinh^{-1} x \equiv \ln(x + \sqrt{x^2 + 1}) \qquad [4]$$

(ii) Find the exact solutions of

$$\cosh^2 x = 9 + 2 \sinh x$$

giving your answers in logarithmic form. [5]

9

Using partial fractions, show that

$$\int_0^1 \frac{x + 3}{(x + 1)(x^2 + 4x + 5)} \, dx = \frac{1}{2} \ln 2 \qquad [10]$$

10

(i) Using partial fractions, show that

$$\frac{7x + 4}{(1 - 3x^2)(2 + 3x)} \equiv \frac{2x + 1}{(1 - 3x^2)} + \frac{2}{(2 + 3x)} \qquad [5]$$

(ii) Hence find a series expansion for

$$\frac{7x + 4}{(1 - 3x^2)(2 + 3x)}$$

up to and including the term in x^3 [5]

(iii) Find the range of values of x for which this expansion is valid. [3]

11

(i) Differentiate

$$\sqrt{9-x^2}$$ [1]

Consider the integral

$$I_n = \int \frac{x^n}{\sqrt{9-x^2}}\,dx \qquad n \geqslant 0$$

(ii) Show that

$$nI_n = -x^{n-1}\sqrt{9-x^2} + 9(n-1)I_{n-2} \qquad n \geqslant 2$$ [7]

(iii) Hence, find the exact value of

$$\int_0^{1.5} \frac{x^4}{\sqrt{9-x^2}}\,dx$$ [7]

12

(i) I represents a measure of electric current after a time t.
I can be modelled by the differential equation

$$\frac{d^2I}{dt^2} + 5\frac{dI}{dt} + 6I = 2\cos t - \sin t$$

Find the general solution of this equation. [9]

(ii) If when $t = 0$ $I = 0$ and $\frac{dI}{dt} = \frac{1}{2}$ show that

$$10I = 2e^{-3t} - 5e^{-2t} + 3\cos t + \sin t$$ [4]

(iii) Using **(ii)** deduce that, as time increases, the measure of current is approximated by the periodic function

$$\frac{1}{\sqrt{10}}\cos(t-\alpha) \qquad \text{where } \tan\alpha = \frac{1}{3}$$ [4]

Total 109 marks Grade A 92/116 = 79%

Grade C 61/116 = 53%

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