Support Material

Unit 1

GCSE
Further Mathematics

UNIT 1

Resources- Unit 1	**Activity**	**Specification Learning Outcome**
1. Equivalent Algebraic Fractions	Card Game	add, subtract, multiply and divide rational algebraic fractions with linear and quadratic numerators and/or denominators
2. Finding LCD	Pair work	algebraic manipulation
3. Expanding three brackets	Worksheet	manipulate algebraic expressions, including the expansion of three linear brackets;
4. Completing the square	Group work to explore the properties of quadratic equations**	complete the square where the coefficient of 2 will always be 1, solving quadratic equations and identifying minimum turning points
5. Solving Quadratic Inequalities	Worksheet**	solve quadratic inequalities, which are restricted to
quadratic expressions that factorise;		
6. Drawing trig graphs	Worksheet	sketch the graphs of sin χ, cos χ and tan χ
7. Equation of a Tangent and Normal to a curve	Worksheet in which learners provide explanations	apply differentiation to finding equations of tangents and normals at points on a curve
8. Integration to find the area under a curve.	Worksheet in which learners provide explanations	evaluate definite integrals and apply integration to finding the area under a curve;

** These two learning outcomes are also supported by PowerPoint files found under the Support tab on the GCSE Revised Further Mathematics (Sept 2017) website

http://www.rewardinglearning.org.uk/microsites/mathematics/revised_further_gcse/support/index.asp U1 R4 Completing the Square for Quadratic Expressions

http://www.rewardinglearning.org.uk/microsites/mathematics/revised_further_gcse/support/index.asp

U1 R6 Solving Quadratic Inequalities

Unit 1: Resource 1: Equivalent Algebraic Fraction Families

Card Game

Suggested approach

Give a card to each pupil in the class. Allow pupils to move around the room to find the other members of their "equivalent fraction family". There are seven families each with four members. When all the members of a family have been found, one member of the group should write the members of that family (the equivalent fractions) on the board.

Alternatively, the whole set of cards can be given to pairs or small groups who can sort through them and group them into "equivalent fraction families."

For pupils who finish the activity quickly, they may enjoy creating their own equivalent fraction families.

This activity could be used to create a wall display by asking pupils to produce a poster showing the different algebraic fraction families and explaining why the members of a particular family are equivalent fractions.

This activity should be used to allow pupils to work together and to encourage whole class discussion on how to find equivalent algebraic fractions.

Reviewing and extending learning

- This work links with simplifying algebraic fractions and can lead to finding equivalent fractions with the same denominator in order to add and subtract algebraic fractions.

$\frac{3y}{z}$	$\frac{6x}{3x+7}$
$\frac{3by}{bz}$	$\frac{4}{2x-4}$
$\frac{6y}{2z}$	$\frac{2}{y}$
$\frac{1}{6}$	$\frac{6x^2}{3x^2+7x}$
$\frac{12ay}{4az}$	$\frac{6y^2}{3y^3}$
$\frac{2}{x-2}$	$\frac{3x-12}{3x}$
$\frac{4x}{2x(x-2)}$	$\frac{6}{8x^3-10}$

$\frac{x(x-3)}{6x(x-3)}$	$\frac{2x(x+1)}{(x+\frac{7}{3})(x+1)}$
$\frac{4(x-3)^2}{24(x-3)^2}$	$\frac{12x}{16x^4-20x}$
$\frac{x-4}{x}$	$\frac{36x}{18x+42}$
$\frac{3x^2}{4x^5-5x^2}$	$\frac{x^3-4x^2}{x^3}$
$\frac{x-3}{6(x-3)}$	$\frac{2x}{x^2-2x}$
$\frac{2xz}{xyz}$	$\frac{y(x-4)}{xy}$
$\frac{12x^2}{16x^5-20x^2}$	$\frac{2x+4}{xy+2y}$

Answers

$$\frac{3y}{z} = \frac{3by}{bz} = \frac{6y}{2z} = \frac{12ay}{4az}$$

$$\frac{6x}{3x+7} = \frac{36x}{18x+42} = \frac{6x^2}{3x^2+7x} = \frac{2x(x+1)}{(x+\frac{7}{3})(x+1)}$$

$$\frac{2}{y} = \frac{2xz}{xyz} = \frac{6y^2}{3y^3} = \frac{2x+4}{xy+2y}$$

$$\frac{x-3}{6(x-3)} = \frac{1}{6} = \frac{4(x-3)^2}{24(x-3)^2} = \frac{x(x-3)}{6x(x-3)}$$

$$\frac{2x}{x^2-2x} = \frac{2}{x-2} = \frac{4}{2x-4} = \frac{4x}{2x(x-2)}$$

$$\frac{3x-12}{3x} = \frac{x-4}{x} = \frac{x^3-4x^2}{x^3} = \frac{y(x-4)}{xy}$$

$$\frac{12x^2}{16x^5-20x^2} = \frac{3x^2}{4x^5-5x^2} = \frac{12x}{16x^4-20x} = \frac{6}{8x^3-10}$$

Unit 1: Resource 2: Finding the Lowest Common Denominator

Suggested approach

Pupils work together in pairs. One person in each pair picks two denominators from the set and the other person has to select the lowest common denominator. Then they switch roles. The winning pair is the pair who can record the most answers in the table provided.

Some pupils may enjoy creating their own examples, that is, one pupil suggests two denominators and his/her partner gives the lowest common denominator.

This activity should be used to allow pupils to work together and to address how to find the lowest common denominator given two denominators.

Reviewing and extending learning

- This work leads to finding the lowest common denominator in order to add and subtract algebraic fractions.

$p^2 - p$	$(x + 2)(x - 1)$
$2a - 3$	$6xy$
$y + 1$	$2y$
$2x$	p
$15x^2$	$(y + 1)(y - 1)$
$6y$	$3y$
5	y
$(y + 1)(y - 1)$	$(2a + 3)(2a - 3)$
$3x^2$	$(x + 2)$

$y-1$	$5x^2$
$3x$	$5x$
$6xy$	$15x$
$6x$	$x-1$
3	$2a+3$
$p-1$	x
$(x+2)(y-1)$	$15x^2y$

Denominator	Denominator	Lowest common denominator
E.g. $3x^2$	$5x^2$	$15x^2$
$(2a+3)$	$(2a-3)$	$(2a-3)(2a+3)$
$(2a-3)$	$(2a-3)(2a+3)$	$(2a-3)(2a+3)$
$(2a+3)$	$(2a-3)(2a+3)$	$(2a-3)(2a+3)$
p	$p-1$	p^2-p
$2x$	$3y$	$6xy$
$5x^2$	3	$15x^2$
$3x^2$	5	$15x^2$
$2x$	3	$6x$
$6y$	$3y$	$6y$
$(y-1)$	$(y-1)(y+1)$	$(y-1)(y+1)$
$(y+1)$	$(y-1)(y+1)$	$(y-1)(y+1)$
3	y	$3y$
$2y$	$3y$	$6y$
x	$5x$	$5x$
$(x+2)$	$(x-1)$	$(x+2)(x-1)$
$(x+2)$	$(y-1)$	$(x+2)(y-1)$
$(y-1)$	$(y+1)$	$(y-1)(y+1)$
$3y$	$15x^2$	$15x^2y$
$2y$	$3x$	$6xy$
3	$5x$	$15x$
$3x$	5	$15x$
$6x$	$6y$	$6xy$
$2x$	$3y$	$6xy$
$2y$	$3x$	$6xy$
3	$6y$	$6y$

These are some possibilities but pupils may find others!

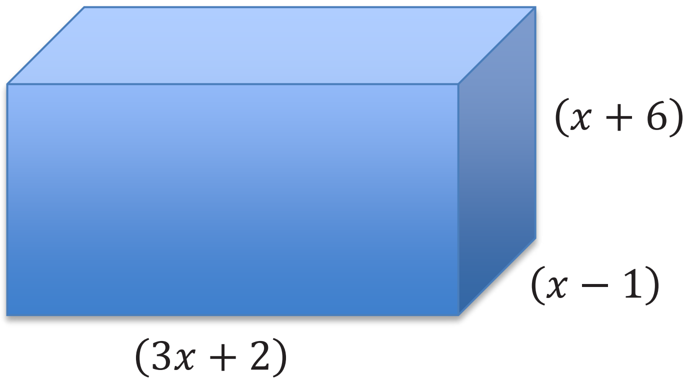
Denominator	Denominator	Lowest common denominator
E.g. $3x^2$	$5x^2$	$15x^2$

Unit 1: Resource 3: Expanding Three Brackets

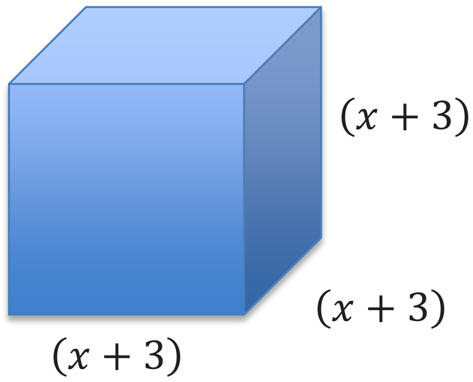
1 Expand and simplify the following.

(a) $(x+3)(x+2)(x+1)$
(b) $(y+5)(y-4)(y+2)$
(c) $(2x+1)(x+6)(x-7)$
(d) $(x-3)(x-2)(4x+1)$
(e) $(y-4)(2y+2)(3y-3)$
(f) $(5x+3)(5x-5)(2x-2)$
(g) $(p-9)(3p-8)(p-2)$
(h) $(x-2)(2x+3)^2$
(i) $(5x-1)^2(4x-1)$
(j) $(4w+5)^3$
(k) $(2m-8)^3$

2 Form an expression for the volume of this cuboid.
Expand and simplify the expression.



3 Form an expression for the volume of this cube.
Expand and simplify the expression.



4 Given that $x^3 + qx^2 + 48x + 64 \equiv (x+p)^3$, find the values of p and q.

5 Given that $x^3 - 21x^2 + ax - 343 \equiv (x-b)^3$, find the values of a and b.

6 Expand and simplify the following.

(a) $(x + y)^0$
(b) $(x + y)^1$
(c) $(x + y)^2$
(d) $(x + y)^3$

Using the coefficients of the terms of your simplified expressions complete the following:

$(x + y)^0$: □

$(x + y)^1$: □ □

$(x + y)^2$: □ □ □

$(x + y)^3$: □ □ □ □

This triangle is known as ____.

Write out the next two lines of Pascal's triangle.

____.

Answers

1

(a) $x^3 + 6x^2 + 11x + 6$
(b) $y^3 \quad + 3y^2 - 18y - 40$
(c) $2x^3 - x^2 - 85x - 42$
(d) $4x^3 - 19x^2 + 19x + 6$
(e) $6y^3 - 24y^2 - 6y + 24$
(f) $50x^3 - 70x^2 - 10x + 30$
(g) $3p^3 - 41p^2 + 142p - 144$
(h) $4x^3 + 4x^2 - 15x - 18$
(i) $100x^3 - 65x^2 + 14x - 1$
(j) $64w^3 + 240w^2 + 300w + 125$
(k) $8m^3 - 96m^2 + 384m - 512$

2 Volume = $(3x + 2)(x - 1)(x + 6)$
= $3x^3 + 17x^2 - 8x - 12$

3 Volume = $(x + 3)^3$
= $x^3 + 9x^2 + 27x + 27$

4 $p = 4$ and $q = 12$

5 $a = 147$ and $b = 7$

6

$$(x+y)^0 = 1$$
$$(x+y)^1 = x+y$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$(x+y)^0$:				1				
$(x+y)^1$:			1		1			
$(x+y)^2$:		1		2		1		
$(x+y)^3$:	1		3		3		1	

This is known as Pascal's Triangle.
The next two lines are 1 4 6 4 1 and 1 5 10 10 5 1.

Unit 1: Resource 4: Completing the Square

Suggested approach

Explain to pupils that there are nine sets of cards each made up of a quadratic function written in different forms, some associated properties and a sketch of the quadratic function. Pupils should work in pairs or small groups to group the cards into the nine sets.

For pupils who are likely to find the activity too easy, you could give them sets of cards but with one missing and they are to create the cards that are missing. They may even enjoy devising their own set of cards.

For pupils who are likely to find the activity too difficult, they could use graphical calculators or graph-drawing software to help them.

This activity could be used to create a wall display by asking pupils to produce a poster of the different sets of cards and writing their reasoning around each set.

This activity should be used to promote discussion in small groups and whole class discussion on how cards were grouped together into the nine sets.

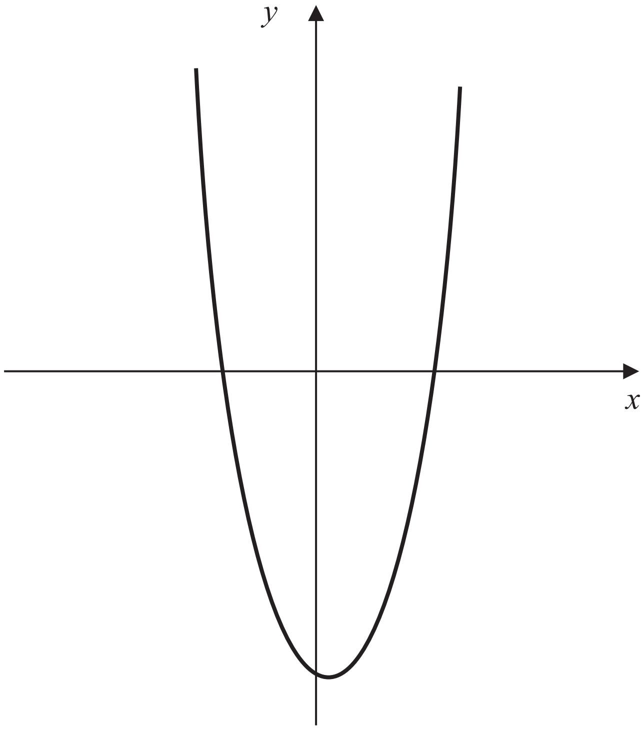
Reviewing and extending learning

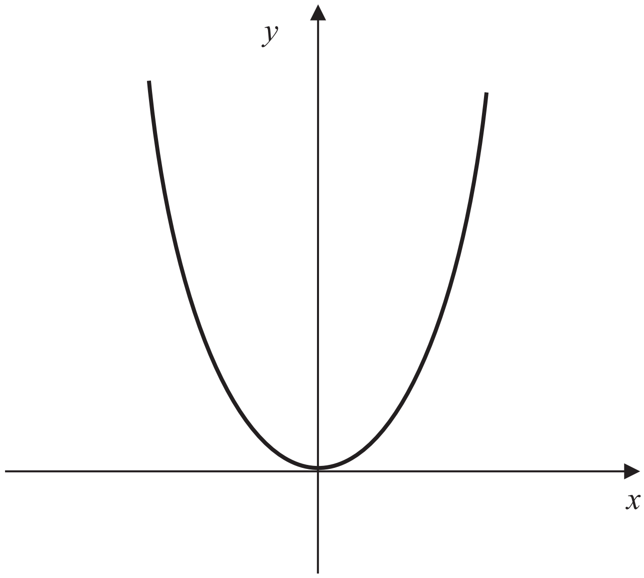
- Pupils could be asked to sketch the graph of a quadratic function, showing clearly any intercepts and the coordinates of the stationary point. When working through the topic of differentiation, make links between using calculus and using the completed square form of a quadratic function to find a stationary point/minimum or maximum value of a curve.
- The completed square form of the quadratic function could be linked to translations of the graphs.

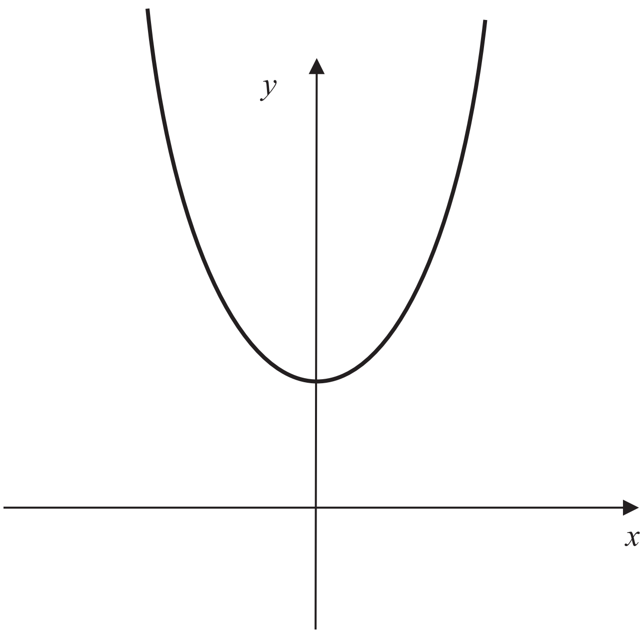
$y = x^2 - 2x - 24$	$y = x^2$
$y = x^2 - 9x + 20$	$y = x^2 - 2x + 1$
$y = x^2 + 11x + 30$	$y = x^2 + 10x + 25$
$y = x^2 + 7$	$y = x^2 + 6x + 11$
$y = x^2 - 8x + 18$	$y = (x - 1)^2 - 25$
$y = (x - \frac{9}{2})^2 - \frac{1}{4}$	$y = (x + \frac{11}{2})^2 - \frac{1}{4}$
$y = (x + 3)^2 + 2$	$y = (x - 4)^2 + 2$
$y = (x + 4)(x - 6)$	$y = (x - 5)(x - 4)$
$y = (x - 1)^2$	$y = (x + 5)(x + 6)$

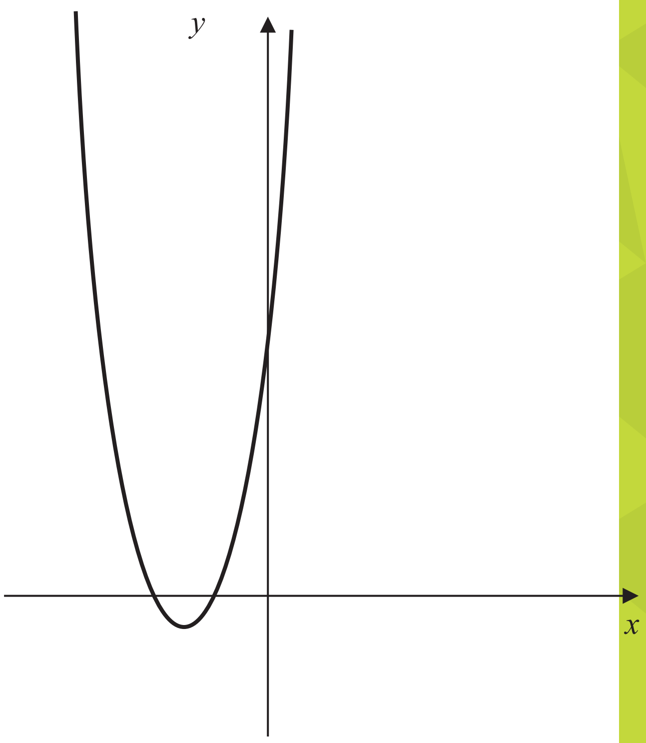
$y = (x + 5)^2$	Minimum at (1, -25)
Minimum at (0, 0)	Minimum at $\left(\frac{9}{2}, \frac{-1}{4}\right)$
Minimum at (1, 0)	Minimum at $\left(\frac{-11}{2}, \frac{-1}{4}\right)$
Minimum at (-3, 2)	Minimum at (4, 2)
x = 0, y = -24	x = 0, y = 0
x = 0, y = 20	x = 0, y = 1
x = 0, y = 30	x = 0, y = 25
x = 0, y = 7	x = 0, y = 11
x = 0, y = 18	y = 0, x = -4 or x = 6

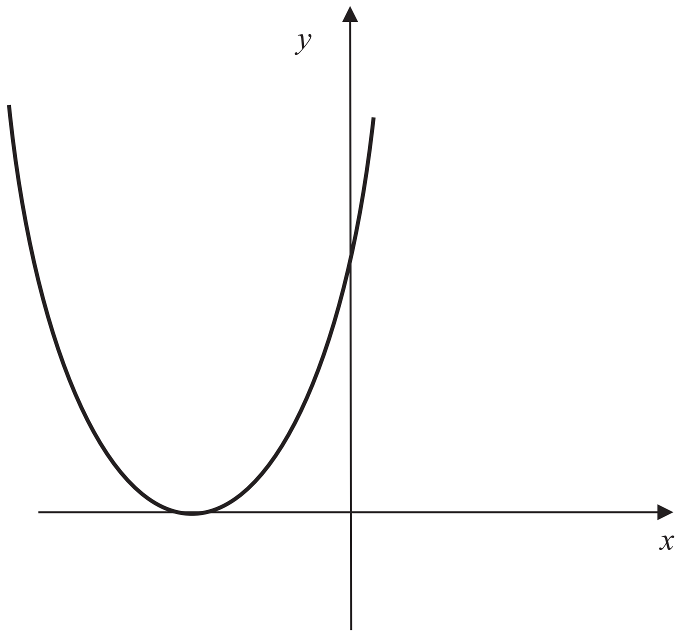
y = 0, x = 5 or x = 4	y = 0, x = 1
y = 0, x = -5 or x = -6	y = 0, x = -5







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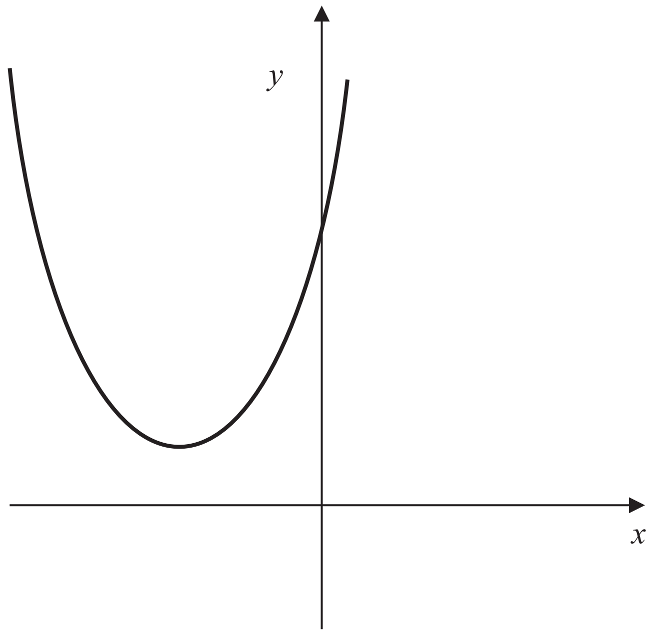
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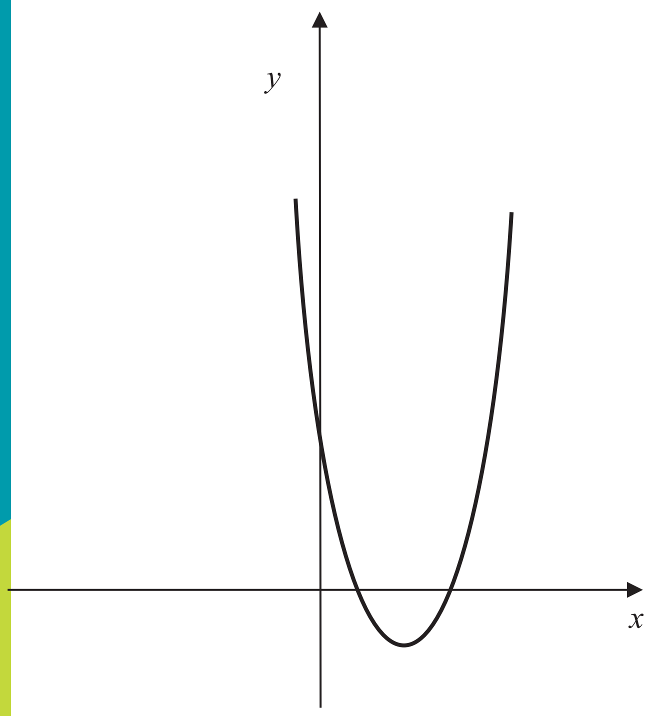
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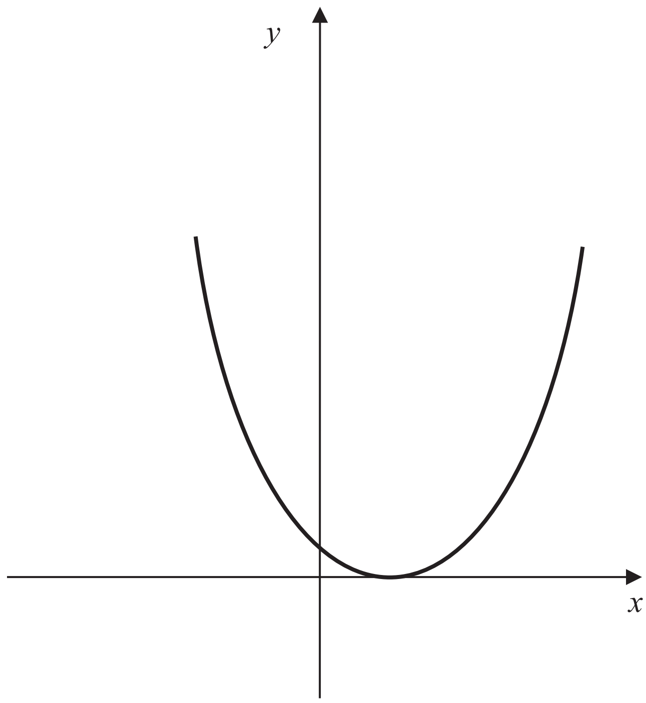
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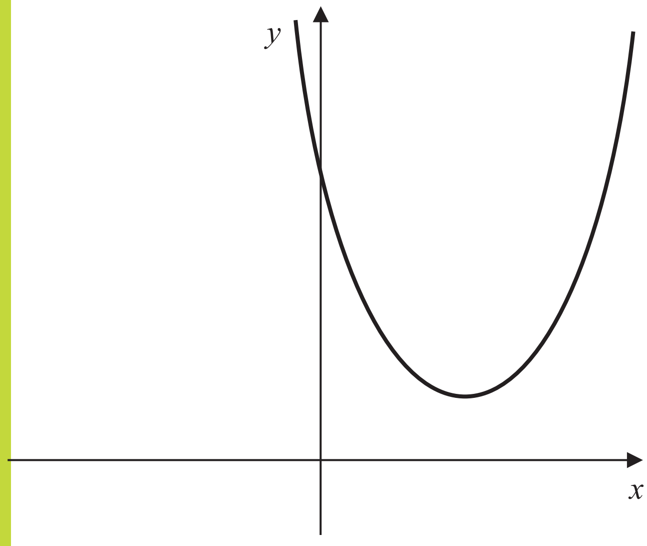

y
x

Unit 1: Resource 5: Solving Quadratic Inequalities

1. Find the set of values of x which satisfy each inequality.

(a) $(x-2)(x+3)>0$
(b) $(x-5)(x+3)\geq 0$
(c) $(x+1)(x+5)>0$
(d) $(x-1)(x-4)<0$
(e) $(2x-1)(x+2)<0$

2. Find the set of values of x for which:

(a) $x^2+7x+12\geq 0$
(b) $x^2-8x+15\leq 0$
(c) $x^2-4x-5\geq 0$
(d) $x^2-11x+24<0$
(e) $12-x-x^2>0$
(f) $2x^2-5x-3\geq 0$
(g) $4x^2-8x+3\leq 0$
(h) $3x^2-19x+6\leq 0$
(i) $6x^2+11x-10>0$
(j) $7+13x-2x^2>0$
(k) $5-3x-2x^2\geq 0$
(l) $x^2-2x<0$
(m) $x^2-5x>0$
(n) $2x^2+3x\leq 0$
(o) $x^2-3x-10>0$
(p) $-2+7x-3x^2<0$
(q) x^2+7x+12≥0
(r) $x^2+5x-6\leq 0$
(s) $10+x-2x^2<0$
(t) $x^2-12x+35<0$
(u) $x^2-9<0$
(v) $3x^2-22x+35\geq 0$
(w) $2x^2+7x+3\geq 0$

3. Determine the range (or ranges) of values x can take for each of the following inequalities.

(a) $6x^2-x>15$
(b) $x^2<10-3x$
(c) $11<x^2+10$
(d) $x(3-2x)>1$
(e) $x^2-1<x+5$
(f) $x^2+8\leq 6x+3$
(g) $4x+1<x^2+4$
(h) $3x^2+2x<1$
(i) $x(x+11)<3(1-x^2)$

Answers

1

(a) $x < -3$ or $x > 2$

(b) $x \leq -3$ or $x \geq 5$

(c) $x > -1$ or $x < -5$

(d) $1 < x < 4$

(e) $-2 < x < \frac{1}{2}$

2

(a) $x \leq -4$ or $x \geq -3$

(b) $3 \leq x \leq 5$

(c) $x \leq -1$ or $x \geq 5$

(d) $3 < x < 8$

(e) $-4 < x < 3$

(f) $x \leq -\frac{1}{2}$ or $x \geq 3$

(g) $\frac{1}{2} \leq x \leq 1\frac{1}{2}$

(h) $\frac{1}{3} \leq x \leq 6$

(i) $x < -2\frac{1}{2}$ or $x > \frac{2}{3}$

(j) $-\frac{1}{2} < x < 7$

(k) $2\frac{1}{2} \leq x \leq 1$

(l) $0 < x < 2$

(m) $x < 0$ or $x > 5$

(n) $-1\frac{1}{2} \leq x \leq 0$

(o) $x < -2$ or $x > 5$

(p) $x < \frac{1}{3}$ or $x > 2$

(q) $x \leq -4$ or $x \geq -3$

(r) $-6 \leq x \leq 1$

(s) $x < -2$ or $x > 2\frac{1}{2}$

(t) $5 < x < 7$

(u) $-3 < x < 3$

(v) $x \leq 2\frac{1}{3}$ or $x \geq 5$

(w) $x \leq -3$ or $x \geq -\frac{1}{2}$

3

(a) $x < -1\frac{1}{2}$ or $x > 1\frac{2}{3}$

(b) $-5 < x < 2$

(c) $x < -1$ or $x > 1$

(d) $\frac{1}{2} < x < 1$

(e) $-2 < x < 3$

(f) $1 \leq x \leq 5$

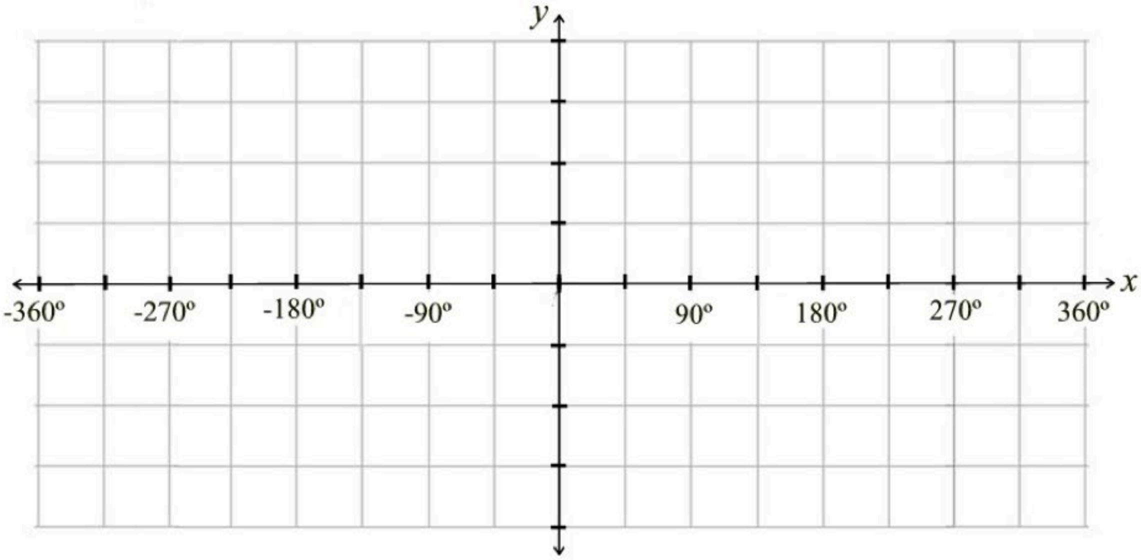
(g) $x < 1$ or $x > 3$

(h) $-1 < x < \frac{1}{3}$

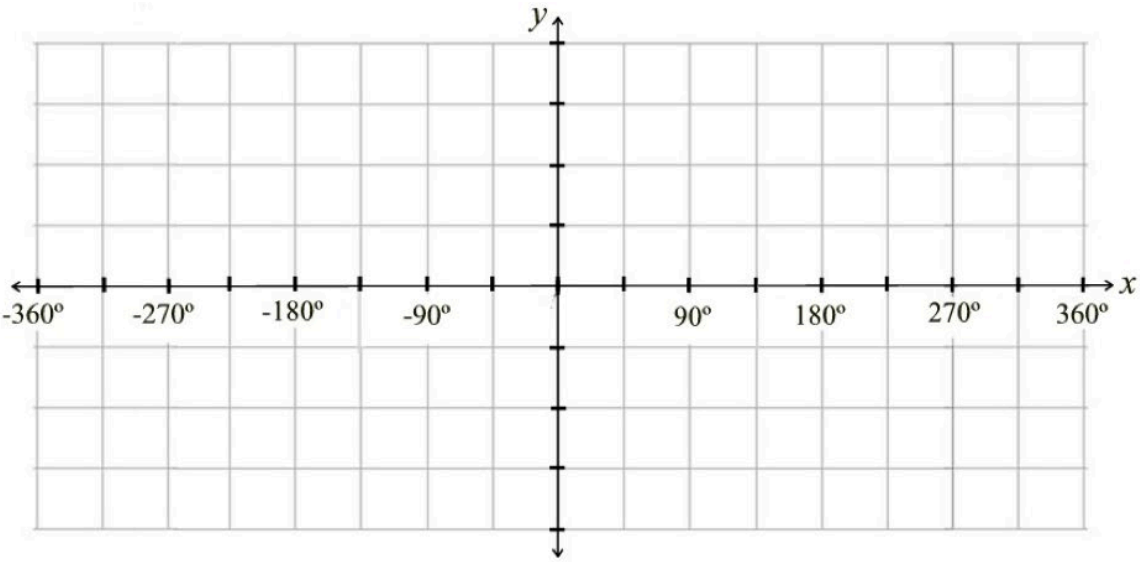
(i) $-3 < x < \frac{1}{4}$

Unit 1: Resource 6: Drawing Graphs of sin x, cos x and tan x

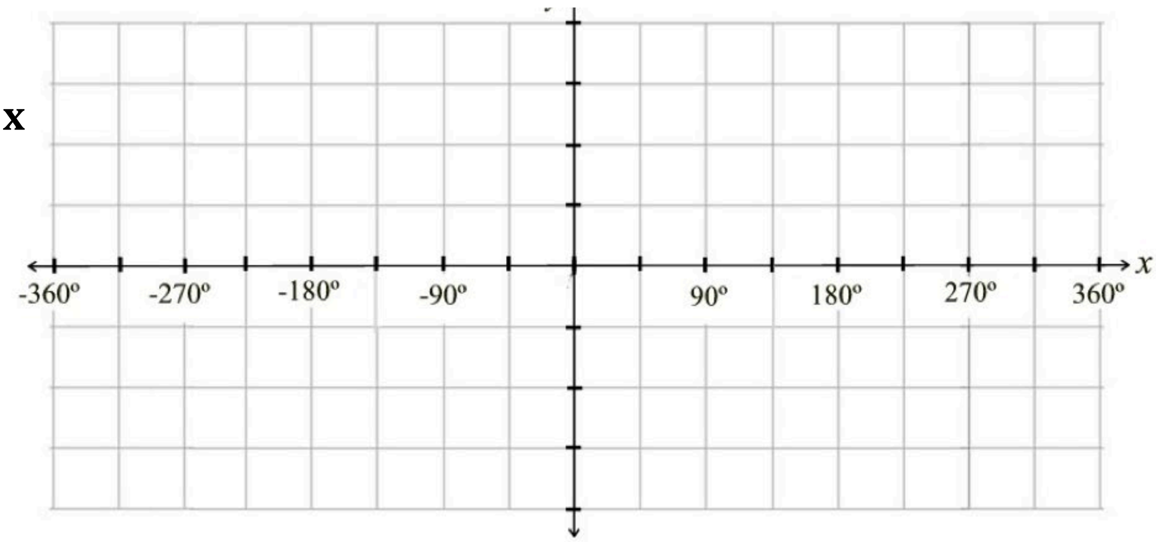
y = sin x



y = cos x



y = tan x



Unit 1: Resource 7: Finding the Equation of a Tangent and Normal to a Curve at a Given Point

Question:

Find the equation of the tangent and the equation of the normal to the curve

$y=5-2x^2$ at the point (-1, 3).

Explain in your own words what is happening at each stage of the solution.

Solution	**Explanation**
$y = 5 - 2x^2$	
$\frac{dy}{dx} = -4x$	
At $(-1, 3), \frac{dy}{dx} = -4(-1) = 4$	
$y = 4x + c$ $3 = -4 + c$ $y = 4x + 7$	
$y = -\frac{1}{4}x + c$ $3 = \frac{1}{4} + c$	
$c = \frac{11}{4}$	
$y = -\frac{1}{4}x + \frac{11}{4}$	

Unit 1 Resource 8: Integration to Find the Area under a Curve

Question:

Find the area enclosed by the curve $y = x + \frac{2}{x^2}$, the x-axis and the lines $x = 1$ and $x = 3$.

Explain in your own words what is happening at each stage of the solution.

Solution **Explanation**

$$\int_1^3 x + \frac{2}{x^2}\,dx$$

$$\int_1^3 x + 2x^{-2}\,dx$$

$$\left[\frac{x^2}{2} + \frac{2x^{-1}}{-1}\right]_1^3$$

$$\left[\frac{x^2}{2} - \frac{2}{x}\right]_1^3$$

$$\left(\frac{3^2}{2} - \frac{2}{3}\right) - \left(\frac{1^2}{2} - \frac{2}{1}\right)$$

$$\left(\frac{27}{6} - \frac{4}{6}\right) - \left(\frac{3}{6} - \frac{12}{6}\right)$$

$$5\frac{1}{3}$$

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