FACTFILE:
GCSE
Engineering and Manufacturing
3.3.1 Materials and Applications – Young's Modulus

Rewarding Learning

Materials and Applications – Young's Modulus

Learning outcomes

Students should be able to:

- interpret stress/strain graphs and perform calculations using Young's modulus (SI units) using:

$$\text{stress} = \frac{\text{force}}{\text{cross-sectional area}}$$

$$\text{strain} = \frac{\text{change in length}}{\text{original length}}$$

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}}$$

- interpret load/extension graphs and stress/strain graphs;
- explain and interpret the elastic limit; and
- explain what is meant by the term Factor of Safety and discuss issues associated with the Factor of Safety.

Explaining and interpreting Elastic limit

The definition of elastic limit can be read as the maximum pressure (stress) that a piece of material can withstand without being permanently deformed. In practical terms it is the maximum extent to which a solid may be stretched without permanent alteration of size or shape.

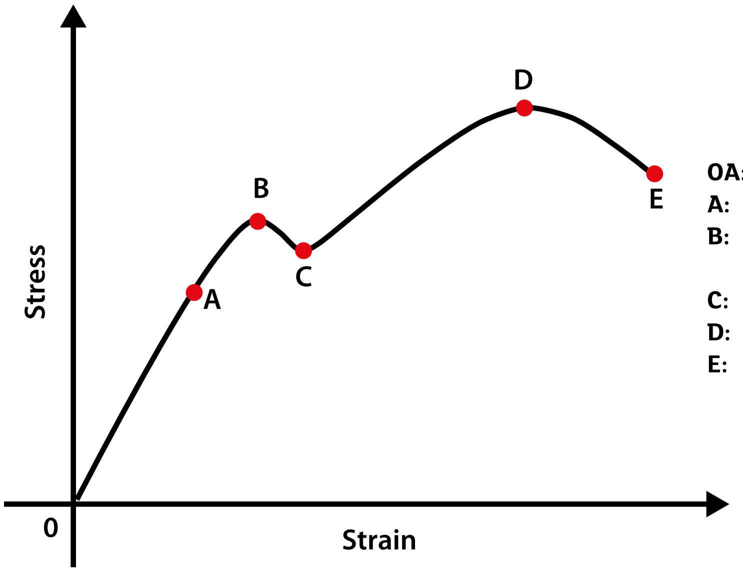
When stresses up to the elastic limit are removed, the material resumes its original size and shape. Stresses beyond the elastic limit cause a material to yield or flow. For such materials the elastic limit marks the end of elastic behaviour and the beginning of plastic behaviour. For most brittle materials, stresses beyond the elastic limit result in fracture with almost no plastic deformation.

SI units

For the purpose of consistency in calculations, all units are expressed in the form of SI (or metric) units. SI units are a system of physical units based on the metre, kilogram and second.

Interpreting a stress/strain graph

The stress-strain curve illustrates the mechanical properties of materials, such as steel, stainless steel or aluminium. The graph can be used to read off by how much the material can stretch in proportion to an increasing applied force. The graph shows how much the material extends under the increasing stress. In order to understand this graph properly, it is important to understand several technical terms.



OA: Proportional limit
A: Elastic limit
B: Yield stress point/upper yield stress point
C: Lower yield stress point
D: Ultimate stress point
E: Breaking or rupture point

Source: http://www.mechanicalbooster.com/2016/09/stress-strain-curve-

In most cases the behaviour of the material can be divided into four phases; these are called:

- elastic deformation;
  The first phase of the stress-strain damage is referred to as elastic deformation. As soon as the stress acting on the material is removed, the material will shorten and return back to its original length. This is called complete recovery or resilience. In this first phase the material stretches proportionally to the stress acting on it.
  This extension can also be referred to as linear-elastic or proportional deformation.

- plastic deformation/flow region (yielding);
  A further, small increase in stress can be enough to cause the proportional limit to be exceeded. Under this force the material begins to flow and the first plastic deformation occurs.
  The area in which the material flows lies between the upper and lower flow limit. The highest flow point is the point accompanied by an initial, sudden loss of quality and the stress required to continue to elongate (strain) the material reduces immediately and reaches the lowest flow point.
  After these points are exceeded the material has definitively deformed irreparably.

- plastic deformation/strain hardening.
  If the stress is increased further plastic deformation will continue and the material will harden and eventually break or rupture.

Calculating stress

Stress is “force per unit area” – the ratio of applied force F to cross section area – defined as “force per area”.

$$\text{stress} = \frac{\text{force}}{\text{cross-sectional area}}$$

- stress = stress measured in Newton/m^2 pascals (Pa)
- F = force in newtons (N)
- A = cross-sectional area in m^2
- So if a force of 100 Newtons was applied to an area of 4 square metres the stress would be;

25 Newton / meter^2

$$\text{Stress} = \frac{100 \text{ Newtons}}{4 \text{ metres}^2} = 25 \text{ Newtons/metre}^2$$

- tensile stress – stress that tends to stretch or lengthen the material.
- compressive stress – stress that tends to compress or shorten the material.
- shearing stress – stress that tends to shear the material.

Calculating strain

To calculate the stain placed on a material use the formula below:

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}}$$

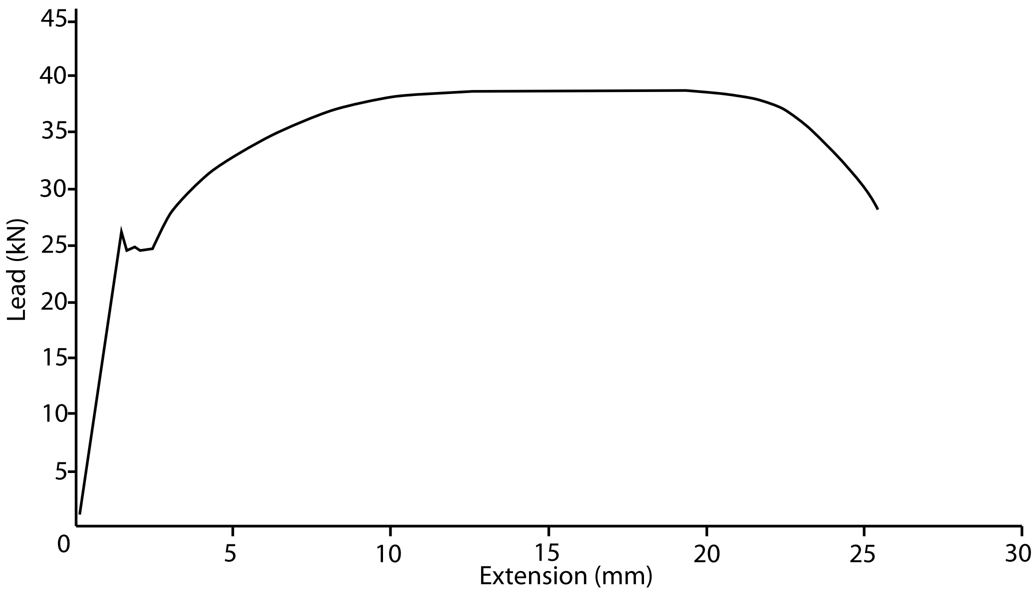
So if the change in length of a 2200 mm steel bar is 12 mm the formula would look like this:

$$\text{Strain} = \frac{12 \text{ mm}}{2200 \text{ mm}} = 0.0054$$

Young's Modulus

Young's modulus, named after the 18th-century English physician and physicist Thomas Young, is a measure of the ability of a material to withstand changes in length when under lengthwise tension or compression. Sometimes referred to as the modulus of elasticity, Young's modulus is equal to the longitudinal stress divided by the strain. Stress and strain may be described as follows in the case of a metal bar under tension. Young's Modulus (E) or the modulus of elasticity is a measure of a materials stiffness. The higher the Young's modulus value the stiffer the material.

Young's modulus can be calculated from tensile test stress/strain graphs–derived from load/extension graphs. The slope of the graph is used to calculate E when the material is obeying Hooke's law.



Source: http://lrrpublic.cli.det.nsw.edu.au/lrrSecure/Sites/Web/tensile_testing

Young's modulus = $\dfrac{\textbf{stress} \text{ (force/cross sectional area)}}{\textbf{strain} \text{ (extension/ original length)}}$

Where stress calculations use units of newtons (N) and metres (m) Young's modulus (E) will be in units of Pascals (Pa)–1Pa = 1 N/m^2. The practical units used are megapascals (MPa or N/mm^2) or gigapascals (GPa or kN/mm^2)

Table: Young's Modulus values for selected materials

Material	Young's Modulus/GPa
Mild Steel	210
Copper	120
Aluminium	69
Plastic	2
Rubber	0.02

(GPa = gigapascal; used in measuring or calculating stresses and pressures).

Source: http://physicsnet.co.uk/a-level-physics-as-a2/materials/young-modulus/

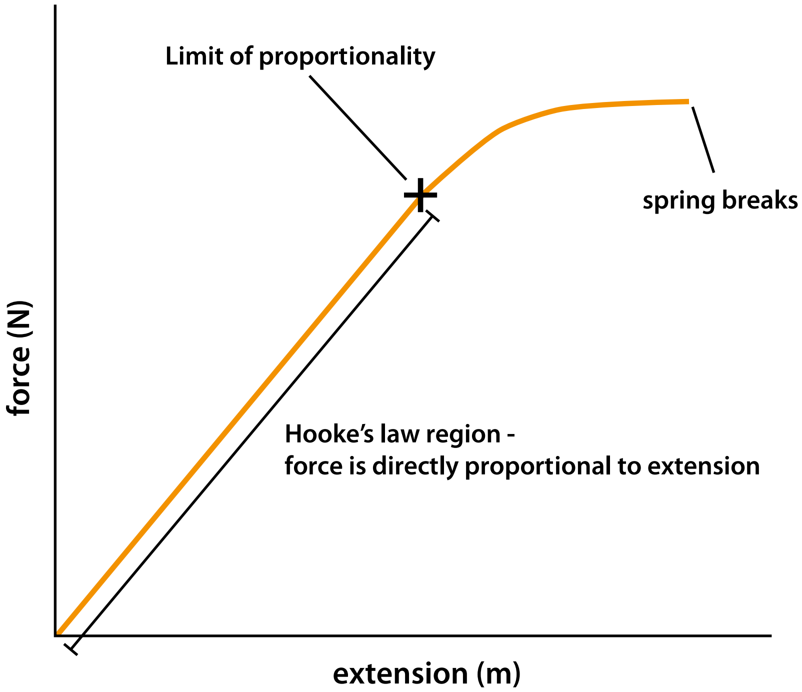
Interpreting load/extension graphs and stress/strain graphs

When an elastic object - such as a spring - is stretched, the increased length is called its extension. The extension of an elastic object is directly proportional to the force applied to it:

Formula is; $F = k \times e$

- F is the force in newtons, N
- k is the 'spring constant' in newtons per metre, N/m
- e is the extension in metres, m

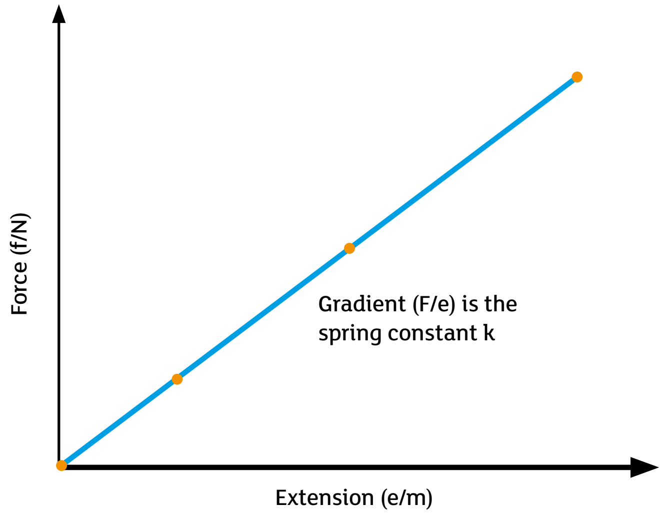
This rule is known as Hooke's Law.



Source: http://physics.taskermilward.org.uk/KS4/additional/forces_and_motion/hooke's_law/

The equation works as long as the elastic limit is not exceeded. If a spring is overstretched, for example, it will not return to its original length when the load is removed. The spring constant **k** is different for different materials.

A graph can be drawn to show the increase in extension with increasing force being applied. A slotted weight is added to the spring and its new length measured. The extension is the new length minus the unloaded length. Additional weights are added and the increase in length is measured. A graph of force against extension can be drawn from the results producing a straight line that passes through the origin. This will be the case as long as the elastic limit is not exceeded.



Source: www.bbc.co.uk/Bitesize/KS3/Science /Physics/Forces and movement

In the graph force is plotted against extension. It would be possible to determine the extension of the spring for any given weight within the lower and upper values of the added weight using the gradient.

The gradient of the line is the spring constant, **k**. The greater the value of **k**, the stiffer the spring.

Factor of Safety

In simple terms, the factor of safety is how much stronger the system is than it usually needs to be for an intended load, it is a multiplier applied to the calculated maximum stress to which a component will be subjected.

The factor of safety (FoS), known also as Safety Factor, is used to provide a design margin over the theoretical design capacity to allow for uncertainty in the design process. This uncertainty could be caused by any one of a number of the elements of the design process. Safety factors are needed to account for imperfections in materials, flaws in assembly, material degradation, and unexpected stresses.

Typically, for components whose failure could result in substantial financial loss, or serious injury or death, a safety factor of at least four (4) is used. Non-critical components generally have a safety factor of two (2). Typical factors of safety used in product design are between 2 and 2.5 for common applications. However, as the factor of safety increases, the cost of the product also increases.

The simplest interpretation of the Factor of Safety is:

$$\text{FoS} = \frac{\text{Actual Breaking Strength of Component}}{\text{Normal Working Load on component}}$$

If a component needs to withstand a load of 100 Newtons and a FoS of 4 is applied then it is designed with strength to support 400 Newtons.

The selection of the appropriate factor of safety to be used in design of components is a compromise between the additional cost and weight and the benefit of increased safety and/or reliability. In general, an increased factor of safety results in a heavier component or a component made from a more expensive material or an improved component design.

General recommendations

The factors of safety listed below are based on the yield strength.

Factor of Safety	Application
1.25–1.5	Material properties known in detail. Operating conditions known in detail. Loads and resultant stresses and strains known with with high degree of certainty. Material test certificates, proof loading, regular inspection and maintenance. Low weight is important to design.
1.5–2	Known materials with certification under reasonably constant environmental conditions. Subjected to loads and stresses that can be determined using qualified design procedures. Proof tests, regular inspection and maintenance required.
2–2.5	Materials obtained for reputable suppliers to relevant standards operated in normal environments. Subjected to loads and stresses that can be determined using checked calculations.
2.5–3	For less tried materials or for brittle materials under average conditions of environment, load and stress.
3–4	For untried materials used under average conditions of environment, load and stress.
3–4	Should also be used with better-known materials that are to be used in uncertain environments or subject to uncertain stresses.

Source: http://www.roymech.co.uk/Useful_Tables/ARM/Safety_Factors.html

Use of Standards and Codes

The use of design codes is a convenient method of ensuring safe confident design. A good standard used by a mechanical engineer is;

BS 2573-Pt 1:1983

Rules for Design of Cranes. Specification for Classification, stress, Calculations and design criteria for structures.

This standard (together with BS 2573 part 2) includes rules for completing calculations and applying factors to be used for the different grades of materials. This standard is primarily used for design of cranes and associated equipment but it is used widely for design of similar mechanical systems. When designing using the standards and codes it is not generally necessary to include additional margins of safety.

When design engineering structures using structural steel section a useful standard is.

BS 5950-1:2000

Structural use of steelwork in building. Code of practice for design. Rolled and welded sections.

Revision Questions

1. Describe what happens to a given material during the elastic deformation stage.

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2. What happens to a material once it has passed the yield point?

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3. Explain why an understanding of the plastic behaviour of engineering materials is essential for the engineers designing structures such as bridges and buildings.

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4. Explain Hooke's Law.

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5. Name the SI unit for pressure.

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6. Name the alternative term sometimes used to describe Young's Modulus.

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7. Describe in simple terms what is meant by Factor of Safety.

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8. In what circumstances would a designer apply a FoS of 3.5 to 4?

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9. Calculate the stress if a force of 20 N acts over a cross-sectional area (csa) of $2m^2$.

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Additional information sources:

www.mathalino.com › Strength of Materials › Chapter 02 – Strain

You Tube: Hooke's Law and the Newton Spring Balance by Professor Mac (https://www.youtube.com/watch?v=zJs27xNdKOM)

You Tube: Hooke's Law Niall Murphy. (https://www.youtube.com/watch?v=WUWMgI438Lg)

http://www.understandingstandards.org.uk/markers_ccc/files/TS_H_ILL_SM4.pdf

http://www.slideshare.net/birendrabiru7/strength-of-maerials-47873792

http://www.instructables.com/id/Steps-to-Analyzing-a-Materials-Properties-from-its/

http://www.engineeringtoolbox.com/stress-strain-d_950.html

http://physicsnet.co.uk/wp-content/uploads/2010/08/stress-1.jpg

http://www.calculatoredge.com/new/stress.htm

https://www.britannica.com/science/elastic-limit

https://www.mechanical360.net/updates/factor-of-safety-and-margin-of-safety/

http://www.engineeringtoolbox.com/

http://lrrpublic.cli.det.nsw.edu.au/lrrSecure/Sites/Web/tensile_testing