



Materials and Applications – Calculations

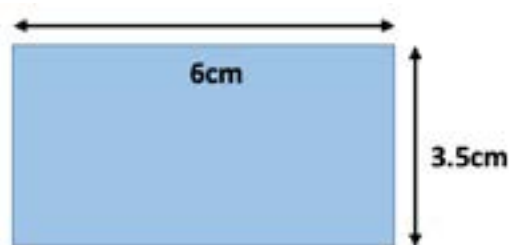
Learning outcomes

Students should be able to:

- use the formulae below to perform calculations in relation to metals (sheet, tubular and round bar), wood and plastic (sheet, tubular and rod):
 - area of a cuboid = length × width ($A = L \times W$);
 - volume of a cuboid = length × width × height ($V = L \times W \times H$);
 - area of a circle = $\pi \times \text{radius}^2$ ($A = \pi r^2$);
 - circumference of a circle = $\pi \times \text{diameter}$ (πD);
 - volume of a cylinder = area × length ($V = A \times L$);
 - area of a triangle = $\frac{1}{2} \text{base} \times \text{height}$ ($A = \frac{1}{2}(B \times H)$);
 - density = mass ÷ volume ($P = \frac{m}{V}$); and
 - percentage = ratio × 100%.

Area of a cuboid

The area of a cuboid is found by multiplying the length by the width. $A = L \times W$



For the cuboid shown above the calculation would be;

$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= 6\text{cm} \times 3.5\text{cm} = 21\text{cm}^2 \end{aligned}$$

The area of the cuboid would be quoted as unit of measurement², for example;

- 21mm²
- 21cm²
- 21m²
- 21km²

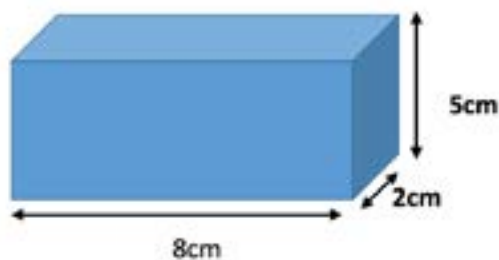
The volume of a cuboid

Volume is a measure of the space taken up by a shape and so is measured in mm³, cm³, etc. Volume can also be measured in litres with 1 litre = 1000cm³.

Do not confuse volume and area.

The volume of a cuboid is found by multiplying length × breadth × height.

$$V = L \times W \times H$$

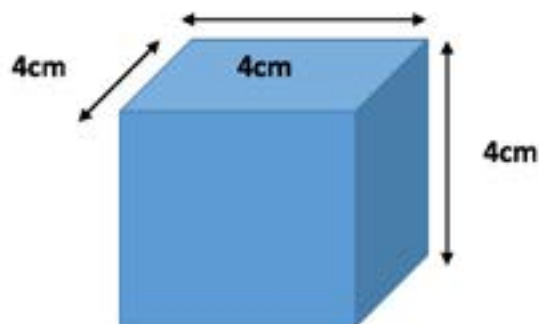


For the cuboid shown above the calculation would be;

$$\text{Volume} = 8\text{cm} \times 5\text{cm} \times 2\text{cm} = 80\text{cm}^3$$

The volume of a cube

For a cube the length, breadth and height are all the same so the volume is found by multiplying length × length × length.



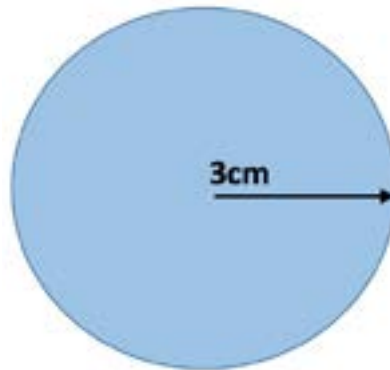
For this cube, Volume = length × length × length.

$$\text{Volume} = 4\text{cm} \times 4\text{cm} \times 4\text{cm} = 64\text{cm}^3$$

The area of a circle

A circumference is a length and is measured in (mm), (cm), (m) etc. Area is measured in square units, (mm²), (cm²) etc. All of these calculations will involve the use of Pi (π) which you can take as being equal to 3.14.

For any circle with radius, r , the area, A , is found using the formula $A = \pi r^2$



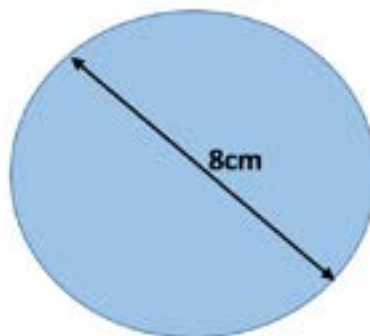
For this circle:

$$A = \pi r^2$$

$$= 3.14 \times 3\text{cm} \times 3\text{cm} = 28.26\text{cm}^2$$

The circumference of a circle.

For any circle with a diameter, D , the circumference, C , is found by using the formula $C = (\pi D)$;



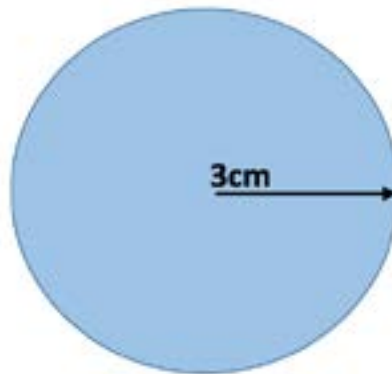
For this circle:

$$C = (\pi D);$$

$$= 3.14 \times 8\text{cm} = 25.12\text{cm}$$

For any circle, the diameter is twice the radius, or $D = 2r$.

If you are given the radius instead of the diameter use the formula $C = 2\pi r$.



For this circle:

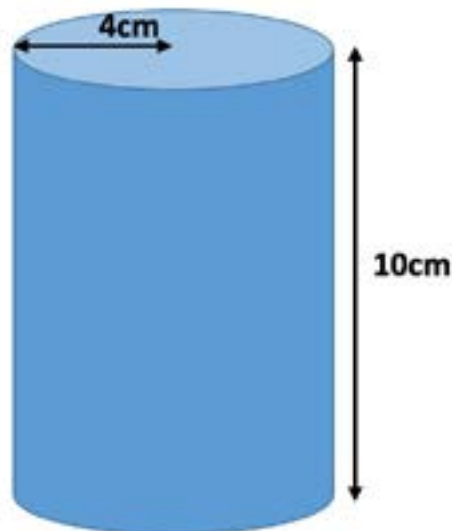
$$C = 2\pi r.$$

$$= 2 \times 3.14 \times 3\text{cm} = 18.84\text{cm}$$

The volume of a cylinder

A cylinder has a circular base and a height. The volume is found by multiplying the area of the base by the length

$$\text{Volume} = A \times L$$



For this cylinder:

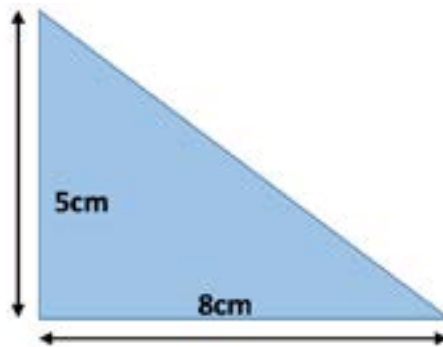
$$A = \pi r^2 \times L$$

$$= 3.14 \times 4\text{cm} \times 4\text{cm} \times 10\text{cm} = 502.4\text{cm}^3$$

Area of a triangle

The area of a triangle is found by multiplying $\frac{1}{2}$ the base by the height.

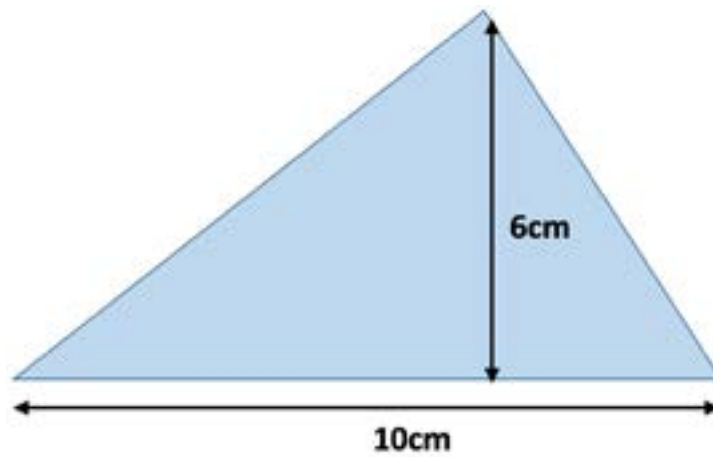
$$A = \frac{1}{2}b \times h$$



For this triangle:

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 8\text{cm} \times 5\text{cm} = 20\text{cm}^2 \end{aligned}$$

For this second triangle:

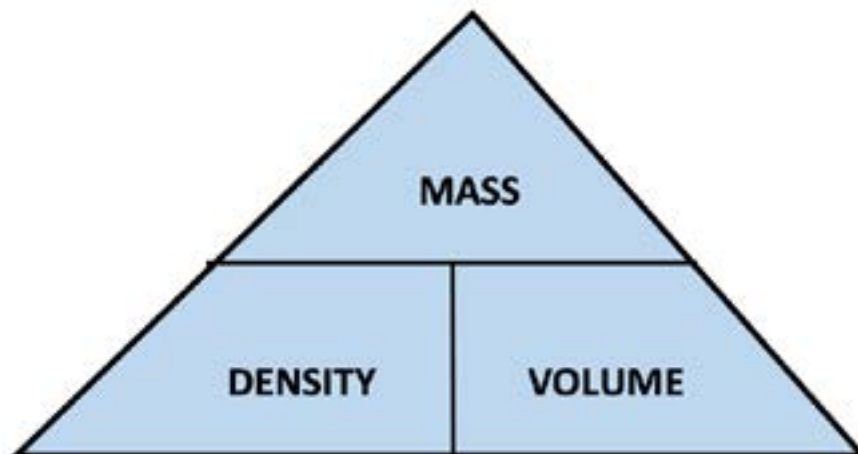


$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 10\text{m} \times 6\text{m} = 30\text{m}^2 \end{aligned}$$

Density

$$(P = m/v)$$

The Density Calculator uses the formula $p=m/V$, or density (p) is equal to mass (m) divided by volume (V). The calculator can use any two of the values to calculate the third. Density is defined as mass per unit volume.



This triangle can help solve a number of calculations in relation to Mass, Volume and Density.

$$\text{Mass} = \text{Density} \times \text{Volume}$$

$$\text{Density} = \text{Mass} \div \text{Volume}$$

$$\text{Volume} = \text{Mass} \div \text{Density}$$

To calculate the density in g/mL of 30 mL of solution that weighs 120 grams use;

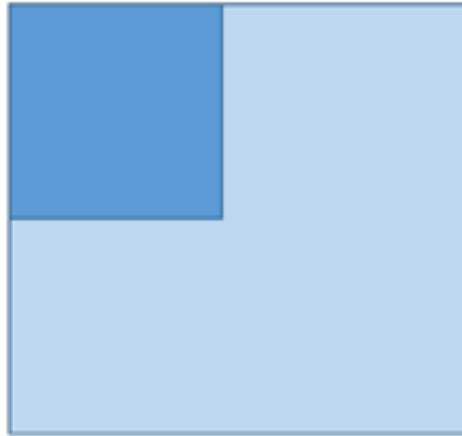
$$\text{Density} = \text{Mass} \div \text{Volume}$$

$$\text{Density} = \frac{120\text{g}}{30\text{mL}}$$

$$\text{Density} = 4\text{g/mL}$$

Percentage = ratio × 100%;

A ratio is another way of comparing quantities. Each quantity must be measured in the same units. An advantage of ratios is that it can compare several things at once.



The ratio of 1: 3 is the ratio of; shaded : unshaded

The ratio of 3: 1 is the ratio of; unshaded : shaded

The ratio of 1: 4 is the ratio of; shaded : whole

The order in which a ratio is written is very important. If the ratio of the number of shaded to the number of unshaded is 1: 3 this is very different to saying the ratio of the number of shaded to the unshaded is 3: 1.

To express the ratio of the shaded to the whole as a percentage, use the formula;

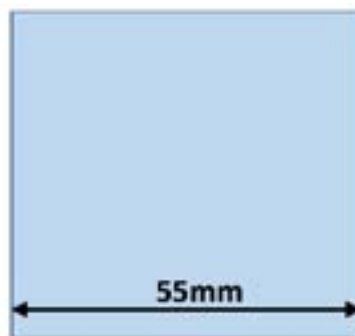
$$\% = \frac{1}{4} \times 100 = 25\%.$$

Revision questions

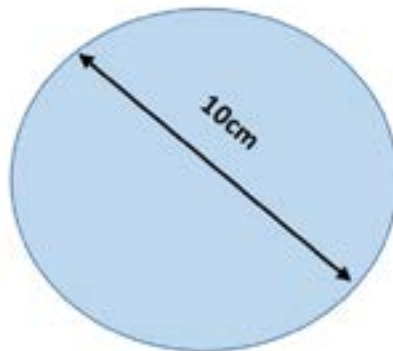
1. Work out the area of this rectangle.



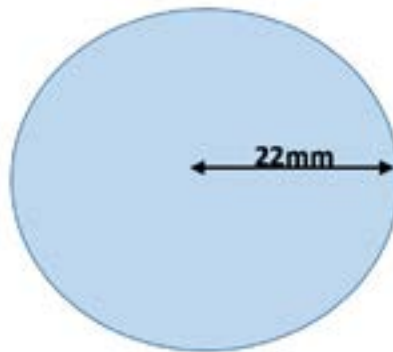
2. Work out the area of this square.



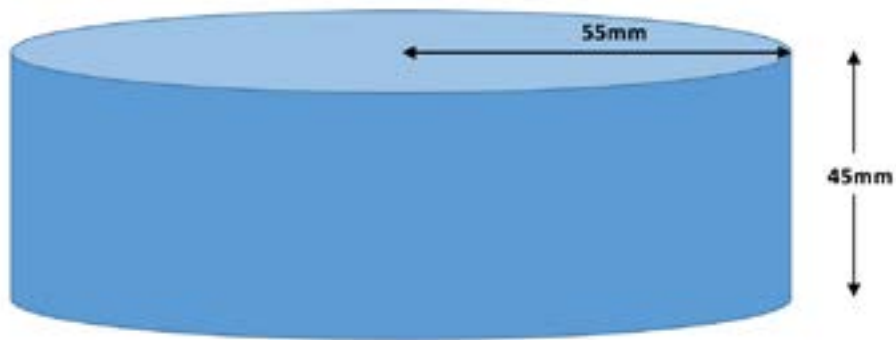
3. Work out the circumference of this circle.



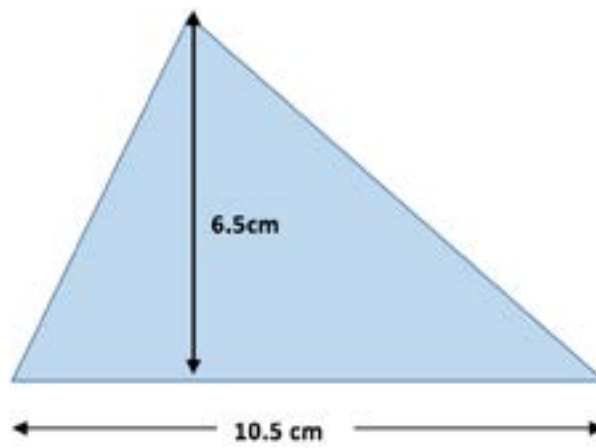
4. Work out the circumference of this circle. Give the answer to 2 decimal places.



5. Calculate the volume of the cylinder shown below.



6. Calculate the area of the triangle below.



7. Calculate the density of a 4kg lump of metal with a volume of 1.25m^3 .

Additional resources:

<https://www.youtube.com/watch?v=YjGVFPtNTHw>

<https://www.youtube.com/watch?v=qJwecTgce6c>

<https://www.youtube.com/watch?v=xCdxURXMdFY>

https://www.youtube.com/watch?v=cC0fZ_lkFpQ

https://www.youtube.com/watch?v=3RIS-yFI0_8

www.bbc.co.uk/schools/gcsebitesize/maths/geometry

<https://www.mansfieldct.org/Schools/MMS/staff/hand/Density.htm>

<https://extranet.education.unimelb.edu.au/SME/TNMY/Arithmetic>

<http://www.mathplanet.com/education/pre-algebra/ratios-and-percent>

