

# FACTFILE: GCE Life & Health Sciences



## BRIDGING MATHEMATICS

This factfile is to help support teachers in 'bridging the gap' from GCSE Mathematics to the mathematics skills required by GCE Life and Health Sciences students.

Teachers should use or adapt this information as required for the units taught, and if necessary, may wish to involve the numeracy coordinators in their school.

There are a number of references to using calculators within this document so it would be beneficial to ensure that all pupils are familiar with using scientific calculators.

### Topics

1. Expressing units in Life and Health Sciences
2. Scientific notation (prefixes and multiples)
3. Converting units
4. Tables
5. Rearranging algebra
6. Ratios, Fractions, Percentages, Decimal Places
7. Graphs (including mapping)
8. Logarithms
9. Exponentials

## 1. Expressing units in Life and Health Sciences

Here are some units you will be familiar with from GCSE:

- velocity is measured in m/s
- acceleration is measured in  $\text{m/s}^2$
- density is measured in  $\text{kg/m}^3$

At A level, equations will start to become more complex, and the units used to express quantities will also be more complex.

For example, how could we find the units for C (specific heat capacity) from the following equation?

$$Q = mC\Delta\theta \quad \text{where} \quad \begin{array}{l} Q \text{ is the energy in joules;} \\ m \text{ is the mass in kg; and} \\ \Delta\theta \text{ is the change in temperature in kelvin K.} \end{array}$$

Rearranging the equation gives:

$$C = \frac{Q}{m\Delta\theta}$$

So the units for C are  $\text{J}/(\text{kgK})$ .

These are not easy units to interpret and so a better way must be considered to express units. We will adopt the rules for expressing indices in mathematics.

$$\frac{1}{x} = x^{-1}, \quad \frac{1}{x^2} = x^{-2}, \quad \sqrt{x} = x^{1/2}, \quad \frac{1}{\sqrt{x}} = x^{-1/2}, \quad \sqrt[3]{x} =$$

$$x^{1/3}, \quad \frac{1}{\sqrt[3]{x}} = x^{-1/3}, \quad \sqrt[3]{x^2} = x^{2/3}, \quad \frac{1}{\sqrt[3]{x^2}} = x^{-2/3}$$

Now apply this to our units above.

$$\text{m/s} = \text{m s}^{-1}, \quad \text{m/s}^2 = \text{m s}^{-2}, \quad \text{kg/m}^3 = \text{kg m}^{-3}$$

Using this notation, the units for specific heat capacity C are  $\text{Jkg}^{-1}\text{K}^{-1}$

### Practice Questions

Try correctly expressing the units for the following quantities:

- Stress (stress = force/area)
- Power ( $P = 1/\text{focal length}$ )
- Specific acoustic impedance ( $z = \text{density} \times \text{speed}$ )
- Decay constant ( $\lambda = 0.693/\text{half-life}$ )
- $T^2$  (time period squared)

### Answers

a)  $\text{Nm}^{-2}$  (also called Pascals Pa); b)  $\text{m}^{-1}$  (also called dioptres D); c)  $\text{kgm}^{-2}\text{s}^{-1}$ ; d)  $\text{s}^{-1}$  (decay constant can also be written  $\text{min}^{-1}$ ,  $\text{hr}^{-1}$ ,  $\text{day}^{-1}$ ,  $\text{yr}^{-1}$  if the units of half life are minutes, hours, days or years); e)  $\text{s}^2$ .

## 2. Scientific Notation (prefixes and multiples)

Expressing numbers in the form  $x^n$  is a common mathematical operation, for example:

$$3 \times 3 \times 3 \times 3 = 3^4 \text{ where } 3 \text{ is the base and } 4 \text{ is the index}$$

Powers of ten can be used to simplify very large numbers, for example:

The speed of light,  $c = 300\,000\,000 \text{ ms}^{-1}$  should be written as  $c = 3 \times 10^8 \text{ ms}^{-1}$ .

Avogadro's constant  $N_A = 602\,300\,000\,000\,000\,000\,000\,000$  should be written as:

$$N_A = 6.023 \times 10^{23}$$

This notation can also be used to express very small numbers, for example:

The resistivity of copper  $\rho = 0.000\,000\,0168 \Omega\text{m}$  is written as  $1.68 \times 10^{-8} \Omega\text{m}$ .

### Prefixes

In science, we often use prefixes to describe the size of a quantity, for example:

A voltage of 8 000 V can be written as  $8 \times 10^3 \text{ V} = 8 \text{ kV}$

A current of 0.003 A can be written as  $3 \times 10^{-3} \text{ A} = 3 \text{ mA}$

The following table contains the prefixes that will be important in your A level studies.

Submultiple	Prefix (symbol)	Multiple	Prefix (symbol)
$10^{-2}$	centi (c)		
$10^{-3}$	milli (m)	$10^3$	kilo (k)
$10^{-6}$	micro ( $\mu$ )	$10^6$	mega (M)
$10^{-9}$	nano (n)	$10^9$	giga (G)
$10^{-12}$	pico (p)	$10^{12}$	tera (T)

### Practice Questions

- Write down the following quantities in standard form without any prefixes.
  - 470 pm; b) 1.5 kV; c) 50 GPa; d) 45 ns; e)  $1\text{pWm}^{-2}$
- Calculate the following:
  - Resistance ( $\Omega$ ) =  $\frac{15 \text{ mV}}{250 \mu\text{A}}$ ; b) Pressure (Pa) =  $\frac{45 \text{ kN}}{2.5 \times 10^{-6} \text{ m}^2}$ ; c) strain =  $\frac{2.5 \text{ mm}}{8.8 \text{ m}}$

### Answers

- 1 a)  $4.7 \times 10^{-10} \text{ m}$ ; b)  $1.5 \times 10^3 \text{ V}$ ; c)  $5 \times 10^{10} \text{ Pa}$ ; d)  $4.5 \times 10^{-8} \text{ s}$
- 2 a) 60  $\Omega$ ; b)  $1.8 \times 10^{10} \text{ Pa}$ ; c)  $2.84 \times 10^{-4}$

### 3. Converting Units

In science we often have to change from one unit to another. Here are some common examples and methods for changing units:

#### Speed

The speed of a car can be changed from  $\text{kmh}^{-1}$  to  $\text{m s}^{-1}$  as follows:

70 km per hour is 70 x 1 000 metres in one hour

This is 70 000 metres in 3 600 seconds which is  $19.4 \text{ m s}^{-1}$

So the method for changing  $\text{kmh}^{-1}$  to  $\text{ms}^{-1}$  is  $\times \frac{1000}{3600}$

#### Density

The density of a material can be changed from  $\text{kg m}^{-3}$  into  $\text{g cm}^{-3}$  by dividing by 1000. An example to demonstrate this is shown below:

The density of aluminium is 2700 kilograms per metre cubed

This is 2 700 000 grams per metre cubed

$1 \text{ m}^3 = 100 \times 100 \times 100 \text{ cm}^3 = 1\,000\,000 \text{ cm}^3$

This is 2 700 000 grams per 1 000 000  $\text{cm}^3 = 2.7 \text{ g cm}^{-3}$

**To convert from  $\text{kg m}^{-3}$  into  $\text{g cm}^{-3}$ , we just divide by 1000**

Density of liquids is sometimes expressed in  $\text{kg l}^{-1}$  (kilograms per litre).

One litre is  $1000 \text{ cm}^3$  (or 1000 ml)

The density of ethanol is approximately  $0.79 \text{ g cm}^{-3}$

This means  $1 \text{ cm}^3$  of ethanol has a mass of 0.79 g

1 litre of ethanol ( $1000 \text{ cm}^3$ ) has a mass of  $0.79 \times 1000 \text{ g} = 790 \text{ g} = 0.79 \text{ kg}$

So ethanol has a density of  $0.79 \text{ gcm}^{-3}$  or  $0.79 \text{ kgl}^{-1}$

**$1 \text{ gcm}^{-3} = \text{kgl}^{-1}$**

#### Area

The area of cross section of a wire can be expressed  $\text{m}^2$  by first converting the diameter of the wire into meters.

For example:

a) The diameter of a wire is 0.4 mm. This is written  $0.4 \times 10^{-3} \text{ m}$

The equation for area is  $A = \pi d^2/4$  where  $d$  is the diameter of the wire.

So area =  $\pi d^2/4 = 3.14 \times (0.4 \times 10^{-3})^2/4 = 1.26 \times 10^{-7} \text{ m}^2$

**The area of small objects can be changed from  $\text{mm}^2$  or  $\text{cm}^2$  to  $\text{m}^2$  by squaring the conversion factor.**

For example:

a) The area of a cylinder =  $84.9 \text{ cm}^2$ . This is  $84.9 \times (10^{-2})^2 \text{ m}^2 = 84.9 \times 10^{-4} \text{ m}^2$

b) The area of a circle =  $201 \text{ mm}^2$ . This is  $201 \times (10^{-3})^2 \text{ m}^2 = 201 \times 10^{-6} \text{ m}^2$

**Volume**

The same principle applies when changing  $\text{mm}^3$  or  $\text{cm}^3$  to  $\text{m}^3$  when expressing volumes.

**The volume of small objects can be changed from  $\text{mm}^3$  or  $\text{cm}^3$  to  $\text{m}^3$  by cubing the conversion factor.**

For example:

a) The volume of a cylinder =  $2.7 \text{ cm}^3$ . This is  $2.7 \times (10^{-2})^3 \text{ m}^3 = 2.7 \times 10^{-6} \text{ m}^3$

b) The volume of a sphere =  $8.5 \text{ mm}^3$ . This is  $8.5 \times (10^{-3})^3 \text{ m}^3 = 8.5 \times 10^{-9} \text{ m}^3$

The volumes of liquids are often expressed in ml and litres, where:

$1000 \text{ ml} = 1 \text{ litre}$ . Since  $1 \text{ ml} = 1 \text{ cm}^3$  and  $1 \text{ litre} = 1000 \text{ cm}^3$ , there are

**$1 \times 10^6 \text{ cm}^3$  in  $1 \text{ m}^3$  and there will be  $1 \times 10^3$  litres in  $1 \text{ m}^3$ .**

**Practice Questions**

- Calculate the cross-sectional area of a wire of diameter:
  - 2.55 mm, b) 4.23 mm.
- Calculate the volume of a cylinder of length 7.44 cm and diameter 1.35 cm.
- Calculate the density in  $\text{kgm}^{-3}$  of a cuboid with a mass of 2.04 kg, a cross-sectional area of  $45.5 \text{ cm}^2$  and length 50.0 cm.

**Answers**

- a)  $5.11 \times 10^{-6} \text{ m}^2$ ; b)  $4.41 \times 10^{-5} \text{ m}^2$
- $1.06 \times 10^{-5} \text{ m}^2$
- $897 \text{ kgm}^{-3}$

## 4. Tables

In most practical work in science results are recorded in a table. All tables have headings – and the headings identify the quantities being measured and the units being used. The following shows an example of a table which might be used in a titration in chemistry.

	Initial burette reading / cm <sup>3</sup>	Final burette reading / cm <sup>3</sup>	Titre /cm <sup>3</sup>
Rough	0.0	24.5	24.5
1st accurate	24.5	48.0	23.5
2nd accurate	0.0	23.2	23.2

Look carefully at the three headings at the top of the table. There is one for each column of numbers. The heading is separated from the unit by a solidus.

Units and calculations are never shown within the body of the table itself. Notice also that the numbers in the table are all shown to the same number of decimal places – for example the final burette reading for the 1st accurate titration is shown as 48.0 and not simply 48.

### Practice Questions

The table below shows the results of a simple pendulum experiment. It shows at least 9 errors. Can you find all of them?

Length, L of pendulum	Time for 20 oscillations /s	Time for 20 oscillations /s	Average	Period, T/s	T <sup>2</sup>	T <sup>2</sup> L <sup>-1</sup> /
0.250	20.00	20.20	$(20.00 + 20.20) / 2 = 20.10$	1.005	1.01	4.04
0.30	23.80	23.9	$(23.80 + 23.9)/2 = 23.85$	1.1925	1.422	4.74
0.5	40.00	39.90	39.95	1.9975	3.99	7.98

## Answers

Length, L of pendulum No solidus, no unit	Time for 20 oscillations /s	Time for 20 oscillations /s	Average Inappropriate heading, no solidus, no unit Better heading might be Average time for 20 oscillations	Period, T / s	T <sup>2</sup> No solidus, no unit – the correct unit is s <sup>2</sup>	T <sup>2</sup> L <sup>-1</sup> / No unit – the correct unit is s <sup>2</sup> m <sup>-1</sup>
0.250	20.00	20.20	(20.00 + 20.20) / 2 = 20.10 Calculations in table	1.01	1.02	4.04
0.30	23.80	23.9 Only 1 dp, others are 2dp	(23.80 + 23.9)/2 = 23.85 Calculations in table	1.1925	1.422	4.74
0.5	40.00	39.90	39.95	1.9975	3.99	7.98

Numbers in first column are all to different numbers of decimal places. 3dp here is probably most appropriate (to nearest mm), since the numbers suggest the lengths are in metres.

Numbers in the fifth and sixth columns are not all to the same number of decimal places.

### Table Exemplar

Now look at the table below which relates to an experiment to find the focal length of a converging lens.

Object distance u / mm	Image distance v / mm	u <sup>-1</sup> / mm <sup>-1</sup> x 10 <sup>-3</sup>	v <sup>-1</sup> / mm <sup>-1</sup> x 10 <sup>-3</sup>	(u <sup>-1</sup> + v <sup>-1</sup> ) <sup>-1</sup> / mm
401	133	2.49	7.52	100
352	140	2.84	7.14	100
301	150	3.32	6.67	100
249	167	4.02	5.99	100
199	196	5.03	5.10	99
150	300	6.67	3.33	100

Mean value of focal length: 100 mm

Notice:

- all columns are labelled with a unit
- there are no units or calculations within the body of the table
- all initial readings are to the nearest mm (consistent with use of a metre stick)
- intermediate calculations are all to the same level of precision (2 decimal places)
- the results are meaningful (typical lens focal lengths lie between 50 mm and 500 mm)
- the final focal length is to the same level of precision as the experimental readings (nearest mm)

## 5. Rearranging Algebra

You should already be comfortable rearranging algebraic equations, so this section is designed to give you practice rearranging some Life and Health Sciences equations. Here is an example you might already be familiar with:

The equation **speed =  $\frac{\text{distance}}{\text{time}}$**  can be rearranged to make distance or time the subject:

$$\text{distance} = \text{speed} \times \text{time} \quad \text{and/or} \quad \text{time} = \frac{\text{distance}}{\text{speed}}$$

But how do we rearrange formulae?

There are 7 basic rules:

1. Remove any square root signs by squaring both sides of the equation.
2. Get rid of any fractions by multiplying EVERYTHING by the denominators.
3. Multiply out all of the brackets .
4. Collect all x terms (or whatever you are solving for) onto one side of the equals sign and move everything else to the other. Remember if it is positive on one side then it becomes negative on the other.
5. Combine the x terms so the equation becomes "Ax = ...".
6. Divide both sides by A so then the equation becomes "x = ...".
7. If it becomes an x<sup>2</sup> term, then the answer is the ± the square root.

### Worked Examples

Here are 6 equations. Let's see how to rearrange then to make the letter in brackets the subject.

$$\text{i) } Q = m c \Delta T \quad (\text{c})$$

The letter "c" only appears at one side of the equation, there are no fractions, square root signs or brackets, so we start at rule 5.

$$Q = c(m \Delta T)$$

Divide both sides by m ΔT

$$\frac{Q}{m \Delta T} = c$$

So,

$$c = \frac{Q}{m \Delta T}$$

$$\text{ii) } R = \frac{1}{t} \quad (\text{t})$$

There is a single fraction with denominator,  $t$ , so we apply rule 2 by multiplying both sides of the equation by  $t$ .

$$R \times t = \frac{1}{t} \times t$$

Multiplying:  $Rt = 1$

Apply rule 6 by dividing both sides by  $R$ :

$$t = \frac{1}{R}$$

$$\text{iii) } \rho = \frac{RA}{L} \quad (\text{R})$$

Again, there is a single fraction with denominator,  $R$ , so we apply rule 2 by multiplying both sides of the equation by  $L$ .

$$\rho \times L = \frac{RA}{L} \times L$$

Simplifying the RHS:  $\rho \times L = RA$

Divide both sides by  $A$ :

$$\frac{\rho L}{A} = R$$

$$R = \frac{\rho L}{A}$$

$$\text{iv) } T = 2 \pi \sqrt{\frac{m}{k}} \quad (\text{m})$$

To remove the square root signs, we apply rule 1 and square both sides of the equation to give:

$$T^2 = 4\pi^2 \frac{m}{k}$$

Now multiply both sides by  $k$  to remove the fraction:

$$T^2 \times k = 4\pi^2 \frac{m}{k} \times k$$

Simplifying:  $kT^2 = 4\pi^2 m$

Apply rule 6 by dividing both sides by  $4\pi^2$  to give:

$$\frac{kT^2}{4\pi^2} = m$$

Leaving the final answer as:  $m = \frac{kT^2}{4\pi^2}$

$$\text{v)} T^2 = 4\pi^2 \frac{L}{g} \quad (\text{g})$$

Multiply both sides by  $g$  to remove the fraction:

$$T^2g = 4\pi^2 \frac{L}{g} \times g$$

Simplifying:  $T^2g = 4\pi^2L$

Divide both sides by  $T^2$  to give the final answer:

$$g = \frac{4\pi^2L}{T^2}$$

$$\text{vi)} \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (\text{f})$$

This equation has 3 fractions with denominators  $u$ ,  $v$  and  $f$ . The common denominator is  $uvf$ .

Multiplying each term by  $uvf$  gives:

$$\frac{uvf}{u} + \frac{uvf}{v} = \frac{uvf}{f}$$

Simplifying gives  $vf + uf = uv$

Factorising gives  $f(v + u) = uv$

Finally dividing by  $(u+v)$  gives:  $f = \frac{uv}{u+v}$

## Practice Questions

Rearrange the following equations to make the letter in brackets the subject.

$$1. E = \frac{\sigma}{\varepsilon} \quad (\sigma)$$

$$2. E = \frac{F \Delta L}{A L_0} \quad (A)$$

$$3. \sigma = \frac{L}{R A} \quad (R)$$

$$4. \frac{Q}{t} = \frac{k A \Delta L}{d} \quad (d)$$

## Answers

$$1. \sigma = E \varepsilon$$

$$2. A = \frac{F \Delta L}{E L_0}$$

$$3. R = \frac{L}{\sigma A}$$

$$4. d = \frac{k A t \Delta L}{Q}$$

## Finding the Unit

We can easily find an unknown unit if we know an equation in which it occurs.

For example in AS 5, in the context of thermal conductivity,  $k$  you will encounter the equation:

$$\frac{Q}{t} = \frac{(k A \Delta T)}{d}$$

Where:

$Q$  = thermal energy in joules

$t$  = time involved in seconds

$A$  = area of cross section (of a bar) in  $\text{m}^2$

$\Delta T$  = difference in temperature in  $^{\circ}\text{C}$

$d$  = length (of the bar) in  $\text{m}$

We can rearrange the equation to make  $k$  the subject:

$$k = \frac{Q d}{t A \Delta T}$$

So the unit for  $k$  = (unit for  $Q$ ) x (unit for  $d$ )

(unit for  $t$ ) x (unit for  $A$ ) x (unit for  $\Delta T$ )

$$\begin{aligned} &= \frac{\text{Jm}}{\text{s m}^2 ^{\circ}\text{C}} \\ &= \text{J m}^{-1} \text{s}^{-1} ^{\circ}\text{C}^{-1} \end{aligned}$$

and since a  $\text{Js}^{-1}$  is a Watt ( $\text{W}$ );

the unit for  $k$  is the  $\text{W m}^{-1} ^{\circ}\text{C}^{-1}$

## Practice Questions

- The specific acoustic impedance,  $Z$ , of human tissue is defined by the equation  $Z = \rho v$  where  $\rho$  is the density of the tissue and  $v$  the speed of sound in the material. Find the unit in which  $Z$  is measured.
- Some physical quantities like strain ( $\varepsilon$ ) in a wire and intensity reflection coefficient,  $R$ , in human tissues have no units. They are defined by the equations:

$$\varepsilon = \frac{\text{extension of a wire}}{\text{original length of the wire}}$$

$$R = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

where  $Z_1$  and  $Z_2$  are the acoustic impedance in tissues 1 and 2 respectively.

Explain why  $\varepsilon$  and  $R$  have no units.

3. A helical spring has mass  $m$  attached to one end. This produces a force  $F$  in the spring. The mass is then displaced and released causing it to oscillate. The equation below represents the relationship for the periodic time (time for one oscillation)  $T$  of a mass-spring system.  $T = 2\pi \sqrt{\frac{m}{k}}$

Rearrange the equation to make  $k$  the subject and find its units.  
(Note that  $4\pi^2$  is just a number and has no units).

## Answers

1. Unit for  $Z = \text{unit for } \rho \times \text{unit for } v = \text{kg m}^{-3} \times \text{m s}^{-1} = \text{kg m}^{-2} \text{ s}^{-1}$

2. Extension and original length have units of length (metres)

So,  $\varepsilon = \frac{\text{extension of a wire}}{\text{original length of the wire}}$  has no unit

From question 1,  $(Z_2 - Z_1)$  and  $(Z_2 + Z_1)$  both have units of  $\text{kg m}^{-2} \text{ s}^{-1}$

So  $(Z_2 - Z_1)^2$  and  $(Z_2 + Z_1)^2$  both have units of  $\text{kg}^2 \text{ m}^{-4} \text{ s}^{-2}$

So  $\frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$  has units of  $\frac{\text{kg}^2 \text{ m}^{-4} \text{ s}^{-2}}{\text{kg}^2 \text{ m}^{-4} \text{ s}^{-2}}$

Since the numerator and denominator are identical,

$R = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$  has no units.

3. Since  $T = 2\pi \sqrt{\frac{m}{k}}$ , then squaring both sides gives  $T^2 = 4\pi^2 \left(\frac{m}{k}\right)$

Multiplying both sides by  $\frac{k}{T^2}$  gives  $k = 4\pi^2 \left(\frac{m}{T^2}\right)$ .  $4\pi^2$  has no units, so the

unit for  $k = \frac{\text{unit for } m}{\text{unit for } T^2} = \text{kg s}^{-2}$

## 6. Ratios, Fractions, Percentages, Decimal Places

### Ratios

A ratio expresses a relationship between two or more quantities. It can be expressed as a fraction (such as  $\frac{5}{7}$ ) or using a colon (such as 5:7). Usually, a ratio is written in its simplest form. For example, 6:30 is the same ratio as 3:15 or as 1:5.

Often, we have to express a ratio in the form n:1. This is the case when we are asked to determine an empirical formula in chemistry. To do that we divide all the numbers in the ratio by the smallest.

Suppose, for example, a compound contains 3.75 moles of hydrogen, 1.25 moles of phosphorus and 4 moles of oxygen and we asked to find its empirical formula.

Then  $H : P : O = 3.75 : 1.25 : 5$ .

The smallest number in this ratio is 1.25, so we divide all the numbers in the ratio by 1.25.

$H : P : O = \frac{3.75}{1.25} : \frac{1.25}{1.25} : \frac{5}{1.25}$  which simplifies to  $H : P : O = 3 : 1 : 4$

The empirical formula is therefore  $H_3PO_4$ .

### Worked Examples

- Suppose the ratio by mass of carbohydrates to proteins to minerals in a meal is 1200 : 400 : 1. If the mass of carbohydrate is 60 g, calculate the mass of the protein and minerals contained in the meal.

There is 1200 times more carbohydrate than mineral.

so the mass of mineral present is  $\frac{60}{1200} = 0.05$  g

For every 1200 g of carbohydrate, there are 400 g of protein.

so the mass of protein is  $(\frac{60}{1200}) \times 400 = 20$  g

**Be careful with ratios when using quantities in different units.**  
This is illustrated in the following example.

- Of the 20 MJ of chemical energy input into a device 500 kJ is wasted as heat and sound. Calculate the ratio Useful energy : Wasted Energy : Total input energy in its simplest terms.

Useful energy = 20 MJ – 0.5 MJ = 19.5 MJ = 19 500 kJ

Useful energy : Wasted Energy : Total input energy = 19 500 : 500 : 20 000

Dividing by 500 gives Useful energy : Wasted Energy : Total input energy = 39: 1: 40

## Fractions (the traditional way)

Most scientific calculators have the ability to give answers as a fraction, such as  $\frac{5}{24}$  although you can always set your calculator to display a decimal number instead. To convert from a fraction to a decimal, you need to divide the numerator (top number) by the denominator (bottom number):  $\frac{5}{24} = 0.208$  (3 dp). Your calculator will have a button to do this calculation for you, often labelled  $S \Leftrightarrow D$ . You also need to be able to add, subtract, multiply and divide fractions.

## Multiplying fractions

Multiplication is very straightforward. You multiply together the numbers on the top (numerators) and then the numbers on the bottom (denominators).

For example:  $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

We can simplify the fraction by dividing the numerator and denominator by 6 to give  $\frac{1}{2}$ .

If we are working with mixed fractions, we first convert to “top-heavy”, sometimes called “improper” fractions.

For example,  $3\frac{1}{2}$  is three wholes plus a half. Each whole number is 2 halves, so three wholes are six halves.

So,  $3\frac{1}{2}$  is  $\frac{7}{2}$  as a “top-heavy” fraction.

Suppose now we want to calculate  $3\frac{3}{4} \times 1\frac{1}{2}$ . Writing both as “top-heavy” fractions we get:

$$3\frac{3}{4} \times 1\frac{1}{2} = \frac{15}{4} \times \frac{3}{2} = \frac{45}{8} = 5\frac{5}{8}$$

## Dividing fractions

To divide fractions, we invert the divisor (the second fraction) and multiply.

For example:

$$\frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{24}{28}$$

We can simplify the fraction by dividing the numerator and denominator by 4 to give  $\frac{6}{7}$ .

If one or more of the fractions is a mixed number, first write it as a “top-heavy” fraction and then proceed as before.

## Adding and subtracting fractions

It is easy to add or subtract fractions if they have the same denominator.

Example  $\frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$  and  $\frac{7}{9} - \frac{4}{9} = \frac{3}{9} = \frac{1}{3}$

If the denominators are not common, we have a preliminary step to express both fractions in terms of a common denominator.

Example  $\frac{3}{4} + \frac{7}{8}$

Here the denominators are not common. But we can write both of them with a denominator of 8.

$$\frac{3}{4} + \frac{7}{8} = \frac{6}{8} + \frac{7}{8} = \frac{13}{8} = 1\frac{5}{8}$$

### Fractions (the easy way)

In every Life and Health Sciences exam you are expected to be able to use a scientific calculator. Learning how to use a calculator to work out fractions is straightforward.

### Adding simple fractions using a calculator

The instructions below tell you how to enter the fraction  $\frac{3}{4}$  and then add  $\frac{1}{2}$ .

- Look for the fraction button; the symbol  $\frac{\square}{\square}$  will appear on the screen when you press it.
- Press the number 3 button; the 3 is displayed as the numerator.
- Press the 'down' navigation button in the center of the calculator when you are ready to enter the denominator.
- Press the number 4 button.
- Press the 'right' navigation button so the fraction  $\frac{3}{4}$  appears on the screen.
- Press +.
- Enter the fraction  $\frac{1}{2}$  in the same way as you entered  $\frac{3}{4}$ , finishing by pressing the 'right' navigation button.
- Press =.
- The display should show the answer:  $\frac{5}{4}$  or  $1\frac{1}{4}$ .
- Press S $\leftrightarrow$ D to see the answer displayed as a decimal (1.25).

### How do you enter mixed fractions into the calculator?

The instructions below show you how you can calculate  $2\frac{1}{2} \times 3\frac{3}{4}$  on your calculator.

- Find and press the mixed fraction symbol – this is usually above the fraction button; you need to press the shift button, then the fraction button. The symbol  $\frac{\square}{\square}$  should appear on the screen.
- You can now enter a mixed fraction.
- Press the number 2 button, and then press 'right' on the navigation button.
- Press the number 1 button, and then press 'down' on the navigation button.

- Press the number 2 button, and then press 'right' on the navigation button.
- The display now shows  $2\frac{1}{2}$ .
- Press the multiply symbol  $\times$ .
- Enter the mixed fraction  $3\frac{3}{4}$  in the same way as you entered  $2\frac{1}{2}$ .
- Press =.
- The display should show  $9\frac{3}{8}$  or  $\frac{75}{8}$ .
- Press  $S \Leftrightarrow D$  to see the answer displayed as a decimal (9.375).

Once you know how to enter fractions in your calculator, you can add, subtract, multiply and divide them easily.

### Reciprocals - Special Fractions

The reciprocal of a number is 1 divided by that number. So, for example, the reciprocal of 5 is 1 divided by 5, which is the fraction  $\frac{1}{5}$ . A reciprocal is also a number taken to the power of -1. So,  $\frac{1}{5}$  is the same as  $5^{-1}$ . Clearly taking the reciprocal of a reciprocal brings you back to the original number, e.g.  $(5^{-1})^{-1} = 5$ .

The reciprocal button on a calculator is usually marked  $x^{-1}$ .

So, to calculate  $\frac{1}{4} + \frac{1}{5}$  we key into the calculator:  $4 x^{-1} + 5 x^{-1} =$

The display will then show  $\frac{9}{20}$ .

To convert this to a decimal we then tap the button  $S \Leftrightarrow D$  and the display changes to 0.45.

### How to calculate a percentage

- Determine the total or whole amount.
- Divide the number to be expressed as a percentage by the total. (In most cases, you'll divide the smaller number by the larger number).
- Multiple the resulting value by 100.

### Example

A solution contains 218.5 g of water and 11.5 g of common salt.

What percentage (by mass) of the solution is common salt and what percentage is water?

### Solution

The total mass of solution =  $218.5 + 11.5 = 230$  g

The percentage of common salt =  $\frac{11.5}{230} \times 100\% = 5\%$

$$\text{The percentage of water} = \frac{218.5}{230} \times 100\% = 95\%$$

Notice that since the solution contains **only** water and common salt, the two percentages add up to 100%.

### How to calculate the percentage of a quantity.

To do this we multiply the percentage by the quantity.

To enter the percentage into a calculator (such as a Casio FX-83GT), enter the number and SHIFT %.

### Example

In naturally occurring uranium ore, 0.7% of the uranium atoms are  ${}^{235}_{92}\text{U}$  and 99.2% of the atoms are  ${}^{238}_{92}\text{U}$ .

In a consignment of ore there is a total mass of 240 g of uranium.

- Find the mass of  ${}^{235}_{92}\text{U}$  atoms and of  ${}^{238}_{92}\text{U}$  atoms in the consignment.
- Suggest why the two percentages do not add up to 100%.

### Solution

$$\text{(i) mass of } {}^{235}_{92}\text{U atoms} = 0.7\% \text{ of } 240 = \frac{0.7}{100} \times 240 = 1.68 \text{ g}$$

$$\text{mass of } {}^{238}_{92}\text{U atoms} = 99.2\% \times 240 = \frac{99.2}{100} \times 240 = 238.08 \text{ g}$$

- In addition to  ${}^{235}_{92}\text{U}$  and  ${}^{238}_{92}\text{U}$  there are other isotopes of uranium in the consignment.

### Percentage change

- Work out the difference between the two numbers being compared
- Divide the increase by the original number
- Multiply the answer by 100%

### Example

The mass of a boy has changed from 45 kg to 63 kg.

What is the percentage increase in his mass?

### Solution

$$63 - 45 = 18 \text{ g}$$

$$\frac{18}{45} \times 100\% = 40\%$$

In summary: **Percentage increase = (increase ÷ original number) × 100%**

**Percentage Yield**

The amount of product you obtain in a chemical process is known as the yield. When the yield is expressed as a % of the maximum theoretical amount, it is called the percentage yield.

$$\text{percentage yield} = \frac{\text{actual yield (appropriate units)}}{\text{theoretical yield (appropriate units)}} \times 100\%$$

**Example 1**

Calculate the percent yield of a reaction that had a theoretical yield of 3.76 g and an actual yield of 1.45 g.

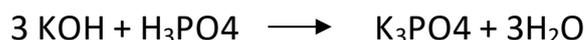
Use the equation:  $\text{percentage yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100\%$

**Solution**

$$\text{percentage yield} = \frac{1.45}{3.76} \times 100\% = 38.6\%$$

**Example 2**

If 2.5 g of  $\text{K}_3\text{PO}_4$  was produced in the reaction below and the percentage yield was 45%, what was the theoretical yield?

**Solution**

Rearrange the equation  $\text{theoretical yield} = \frac{\text{actual yield}}{\text{percent yield}} \times 100$

$$\text{theoretical yield} = \frac{2.5}{45} \times 100 = 5.56 \text{ g}$$

**Example 3**

Find the percentage of calcium in calcium carbonate,  $\text{CaCO}_3$ .

**Solution**

The percentage yield should first be converted from a percentage to a decimal.

Relative formula mass ( $M_r$ ) of calcium carbonate,  $\text{CaCO}_3 = 40 + 12 + (3 \times 16) = 100$

Work out the total relative atomic mass ( $A_r$ ) of all of the atoms of the element.

There is only one atom of calcium in  $\text{CaCO}_3$ , so the total mass of all the calcium atoms

$$= 1 \times 40 = 40$$

The percentage of calcium in  $\text{CaCO}_3 = \frac{40}{100} \times 100\% = 40\%$

## Decimal Places

Often you are required to express a number to 1, 2 or 3 decimal places. Let's look at what this means and how it can be done.

Consider the number 0.6547. This number is to 4 decimal places, because there are 4 integers (whole numbers) after the decimal point. If we are to express a number to 3 decimal places (dp), the 4<sup>th</sup> decimal place is the decider. If we are to express it to 2 decimal places, the 3<sup>rd</sup> decimal place is the decider, and so on. If the decider figure is greater than 4, the number immediately before it is promoted, otherwise it remains unchanged.

To express 0.6547 to 3 decimal places, we inspect the decider figure. In this case the decider is 7 so the 4 is promoted to 5. So, to 3 dp, the number 0.6547 is 0.655.

To express 0.6547 to 2 decimal places, we inspect the decider figure. In this case the decider is 4 so the 5 is not promoted – it stays as 5. So, to 2 dp, the number 0.6547 is 0.65.

To express 0.6547 to 1 decimal place, we inspect the decider figure. In this case the decider is 5 so the 6 is promoted to 7. So, to 1 dp, the number 0.6547 is 0.7.

A difficulty can arise if the number to be promoted is a 9. Consider the number 7.985. This number is already to 3 dp. To 2 dp, it is 7.99 because the decider is 5. But what happens when we express 7.985 to 1 dp? Here the decider is the 8, so the 9 is promoted to 10. This changes the 9 to zero **and** increases the 7 to 8. So, to 1 dp, the number 7.985 is written as 8.0.

Decimal places can be really important in how we interpret data. Suppose a scientist records the diameter of a metal rod as 0.7 cm, to 1 dp. Clearly the rod could have had a diameter just below or just above 0.7 cm, but when the measurement is given to 1 dp, the figure is 0.7 cm.

But, to 2 dp, what are the smallest and largest values of the rod's diameter? The smallest diameter is the smallest figure (to 2dp) which would have been rounded up to 0.7. This is 0.65 cm. The largest diameter is biggest number (to 2 dp) which would be rounded down to 0.7. This is 0.74 cm. So,  $0.65 \text{ cm} \leq \text{diameter} \leq 0.74 \text{ cm}$ , to 2 dp.

## Practice Questions

1. An alloy is made from 57.8 g of copper, 8.16 g of tin and 2.04 g of zinc.  
Calculate the percentage of each metal in the alloy.
2. The ratio of hydrogen to oxygen in a compound is 1:8.  
Find the percentage of oxygen in the compound.  
What is the mass of hydrogen in 200 g of the compound?
3. The nutrition information on the box containing a ready meal claims that the energy content of the meal is 542 kilocalories and that this represents 27.1 % of the guideline daily energy intake for an average adult in the UK.  
Calculate the guideline daily energy intake for an average adult in the UK.
4. An A4 sheet of paper measures 210 mm wide by 297 mm long. Each measurement being recorded to the nearest mm. Write down the maximum and minimum dimensions of the paper, giving your answers in mm to 1 dp.
5. The number shown on a calculator is 0.652652652.... (this is sometimes written as 0.652 with a dot above the numbers 6 and 2 to show that they recur indefinitely).  
Write this number to 1, 2 and 3 dp.
6. A toy car rolls down a slope and travels 20 cm in 6.4 s.  
Calculate its average speed to 2 dp.

## Answers

1. The total mass of the alloy is 68.1 g

$$\text{Percentage copper} = \frac{57.8}{68.1} \times 100 = 85 \%$$

$$\text{Percentage tin} = \frac{8.16}{68.1} \times 100 = 12 \%$$

$$\text{Percentage zinc} = \frac{2.04}{68.1} \times 100 = 3 \%$$

2. Percentage of oxygen =  $\frac{8}{9} \times 100\% = 88.9\%$

$$\text{Percentage of hydrogen} = 100\% - 88.9\% = 11.1\%$$

$$\text{Mass of hydrogen} = \frac{11.1}{100} \times 200 \text{ g} = 22.2 \text{ g}$$

3. 27.1% of guideline amount = 542 kilocalories

$$1 \% \text{ of guideline amount} = \frac{542}{27.1} = 20 \text{ kilocalories}$$

$$100 \% \text{ of guideline amount} = 20 \times 100 = 2000 \text{ kilocalories}$$

4. Max: 210.4 mm x 297.4 mm; Min: 209.5 mm x 296.5 mm

5. 1 dp – 0.7; 2 dp – 0.65; 3 dp 0.653

6. average speed =  $\frac{\text{distance}}{\text{time}} = \frac{20 \text{ cm}}{6.4 \text{ s}} = 3.125 \text{ cms}^{-1} = 3.13 \text{ cms}^{-1}$  (to 2 dp)

## 7. Graphs (including mapping)

Any relationship between two quantities can be plotted as a straight-line plot provided all other quantities in the equation are constant. For example, consider the relationship between the resistance  $R$  of a uniform wire and its length  $L$ .

$$R = \frac{\rho L}{A} \quad \text{where } \rho \text{ is the resistivity of the wire and } A \text{ is the cross-sectional area.}$$

We can map the equation  $R = \frac{\rho L}{A}$  on to the equation of a straight-line  $Y = mx + c$

Where  $y$  is the quantity to be plotted on the  $y$ -axis,  $x$  is the quantity to be plotted on the  $x$ -axis,  $m$  is the gradient of the graph and  $c$  is the  $y$ -intercept.

$$\begin{array}{c} R = \frac{\rho}{A} L \\ \downarrow \quad \downarrow \quad \downarrow \\ y = m x + c \end{array}$$

The quantity to be plotted on the  $y$ -axis is  $R$ , the quantity to be plotted on the  $x$ -axis is  $L$ , the gradient of the graph is  $\frac{\rho}{A}$  and the  $y$ -intercept is 0.

What about the equation the relationship  $T = 2\pi \sqrt{\frac{L}{g}}$  between the time period of a pendulum  $T$  and the length of the pendulum,  $L$  where  $g$  is the acceleration due to gravity?

$$\begin{array}{c} T = \frac{2\pi}{\sqrt{g}} \sqrt{L} \\ \downarrow \quad \downarrow \quad \downarrow \\ y = m \quad x + c \end{array}$$

From mapping the equation, we can see that the quantity to be plotted on the  $y$ -axis is  $T$ , the quantity to be plotted on the  $x$ -axis is  $\sqrt{L}$ , the gradient of the graph is  $\frac{2\pi}{\sqrt{g}}$  and the  $y$ -intercept is 0.

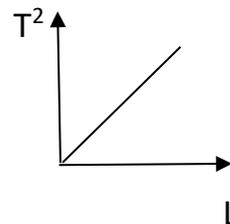
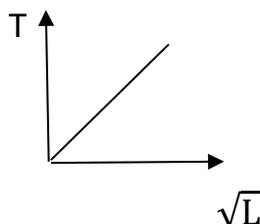
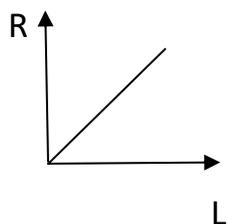
Alternatively, the equation can be squared as follows:

$$\begin{array}{c} T^2 = \frac{4\pi^2}{g} L \quad \text{and this can also be mapped onto the equation of a straight line to give} \\ \downarrow \quad \downarrow \quad \downarrow \\ y = m \quad x + c \end{array}$$

From mapping the equation, we can see that the quantity to be plotted on the  $y$ -axis is  $T^2$ , the quantity to be plotted on the  $x$ -axis is  $L$ , the gradient of the graph is  $\frac{4\pi^2}{g}$  and the  $y$ -intercept is 0.

All these relationships when plotted will produce a straight-line graph which goes

through the origin.



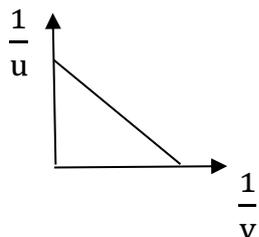
Let's consider a different relationship  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$  which describes the relationship between the object distance from a lens  $u$ , the image distance from the lens,  $v$  and the focal length  $f$ . Since  $u$  and  $v$  are the variables in this relationship, the quantities  $\frac{1}{u}$  and  $\frac{1}{v}$  can be plotted to investigate the relationship. Re-arranging this equation produces the following:

$$\frac{1}{u} = - \frac{1}{v} + \frac{1}{f}$$

↓   ↓   ↓   ↓

$$y = -1 \ x + c$$

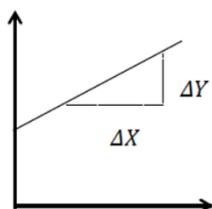
Since there is no number in front of the  $\frac{1}{v}$  term, just a minus sign, then the value of  $m$  the gradient is  $-1$ . Also, you can see the  $y$ -intercept is  $\frac{1}{f}$ .



### Gradients of straight line graphs

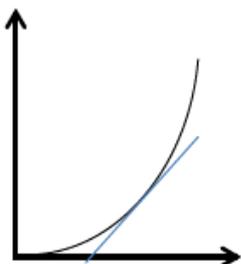
To determine the gradient of a graph we need to use the formula:

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{\Delta Y}{\Delta X}$$



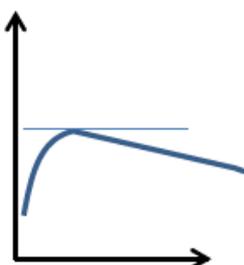
- Choose any two points on the best fit line ( $x_1, y_1$ ) and ( $x_2, y_2$ )
- Subtract the two  $y$  values to obtain  $\Delta y$  and subtract the two  $x$  values to obtain  $\Delta x$ .
- Now divide  $\frac{\Delta Y}{\Delta X}$  to obtain the gradient.
- Always choose two points which are far apart to reduce the error in calculating the gradient.

### Non-linear graphs



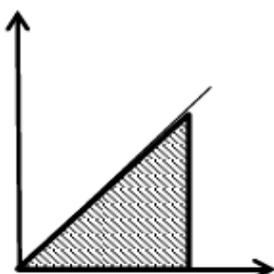
The gradient of this graph is getting steeper. To find the gradient of the graph at any point you must first draw a tangent to the curve at that point and then calculate the gradient of the tangent line.

### Maximum and minimum turning points



At a turning point the tangent of the curve is horizontal and the gradient is zero.

### Area under the graph



Sometimes it is useful to calculate the area under a graph. We can do this by calculating the area of the shape drawn between the line and the x-axis.

## Practice Questions

- Map the following relationships between two variables onto the equation of a straight line. What is the gradient of the line and intercept (if any)?

(i)  $R = \frac{\rho L}{A}$  (R and A);      (ii)  $n = \frac{\sin i}{\sin r}$  (sin i and sin r).

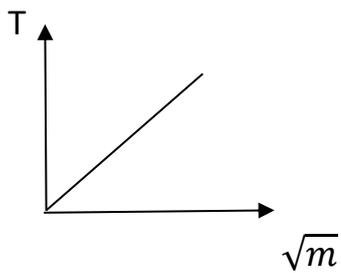
- The equation  $T = 2\pi \sqrt{\frac{m}{k}}$  is an equation which shows the relationship between the time period of a spring T and the mass added to the spring m. Sketch two possible straight-line graphs to show the relationship between T and m.

## Answers

1. (i) Plot  $R$  against  $\frac{1}{A}$ , the gradient will be  $\rho L$  and there will be no intercept.

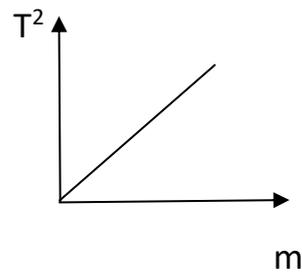
(ii) Plot  $\sin i$  against  $\sin r$ , the gradient will be  $n$ .

2.



$$\text{Gradient} = \frac{2\pi}{\sqrt{k}}$$

or



$$\text{Gradient} = \frac{4\pi^2}{k}$$

## 8. Logarithms

In Life and Health Sciences we often work with powers such as squared and cubed e.g.

Find  $x^2$  when  $x = 4.5$

$$4.5^2 = 4.5 \times 4.5 = 20.25$$

We also work with fractional powers e.g.

Find  $\sqrt{x}$ , written as  $x^{1/2}$

when  $x = 17$

$$x^{1/2} = 17^{1/2} = 4.12$$

An expression such as  $2^3$  can be determined using a calculator.

$$2^3 = 8$$

But what about  $2^n = 16$ ?

The inverse operation would be as follows:

$$\log_2 16 = 4$$

### What are logarithms?

The logarithm or log of a number is the power to which you must raise a base in order to obtain the number. In the example above 16 is the number, 2 is the base and 4 is the power.

$$\text{If } y = x^n \text{ then } \log_x y = n$$

### Log to the base 10

One of the base numbers we work with in Life and Health Sciences is base 10.

Log to the base 10 is written as  $\log_{10}$ .

Try the following two operations using your calculator.

a)  $10^7 = 10\,000\,000$

$$\log_{10} 10\,000\,000 = \log_{10} 10^7 = 7$$

b)  $10^3 = 1000$

$$\log_{10} 1\,000 = \log_{10} 10^3 = 3$$

### Rules for logs

**Rule 1**  $\log(a \times b) = \log a + \log b$

e.g.  $100\,000 = 1\,000 \times 100$

$$\log(100\,000) = \log 1\,000 + \log 100$$

$$\log 10^5 = \log 10^3 + \log 10$$

**Rule 2  $\log(a^n) = n \log a$** 

e.g.  $\log(5^3) = 3 \log 5$

$2.10 = 3 \times 0.699$

**Rule 3  $\log_x x^n = n$** This is because  $\log_x$  is the inverse operation of  $x^n$ 

e.g.  $\log_{10} 10^8 = 8$

An example of log operations in Life and Health Sciences is in the topic of Sound in A24. The reason we use logs to express sound intensities is because sound intensities range from extremely small values,  $1 \times 10^{-12} \text{ Wm}^{-2}$  the threshold of hearing to extremely large values,  $1 \times 10^4 \text{ Wm}^{-2}$  which would result in instant perforation of the eardrum. Since this range of intensities is so large **the decibel scale** is often used to express these numbers and is called the **decibel level**, measured in **decibels, dB**.

The follow equation is used to calculate sound intensity:

$$I = I_0 \times 10^{\frac{\text{db level}}{10}} \quad \text{where} \quad \begin{array}{l} I = \text{sound intensity (Wm}^{-2}\text{)} \\ \text{dB level} = \text{decibel level (sometimes called intensity level)} \\ I_0 = \text{threshold intensity } 1 \times 10^{-12} \text{ Wm}^{-2} \end{array}$$

The equation  $I = I_0 \times 10^{\frac{\text{db level}}{10}}$  can be rearranged to provide an equation that will allow the db level to be calculated. This is done by taking log of each side of the equation.

Using our previous log rules:

$$\log I = \log I_0 + \frac{\text{db level}}{10}$$

$$\log I = \log I_0 + \frac{\text{db level}}{10}$$

$$\text{db level} = 10 \log_{10} \frac{I}{I_0}$$

## Practice Questions

1. The table below shows a range of different sound intensities, in  $\text{Wm}^{-2}$  and intensity levels or decibel levels, in dB. Fill in the missing values.

Source	Intensity/ $\text{Wm}^{-2}$	Decibel level/dB
Whisper		20
Library	$1.2 \times 10^{-10}$	
Vacuum cleaner	$0.98 \times 10^{-4}$	
Orchestra		95
Loud Earphones		98
Front row at concert	$1.02 \times 10^{-1}$	

2. A pupil talks in class with an intensity level (or decibel level) of 60 dB. What is the new decibel level if nine pupils talk at once?
3. A person talks whilst vacuuming. What is the resultant intensity and decibel level (person talking 60 dB, vacuum cleaner, 72 dB)?
4. Express the following intensities in  $\text{pWm}^{-2}$ 
  - a)  $1 \times 10^{-12} \text{ Wm}^{-2}$
  - b)  $3.3 \times 10^{-8} \text{ Wm}^{-2}$

## Answers

1.

Source	Intensity/ $\text{Wm}^{-2}$	Decibel level/dB
Whisper	$1 \times 10^{-10}$	20
Library	$1.2 \times 10^{-10}$	20.8
Vacuum cleaner	$0.98 \times 10^{-4}$	80
Orchestra	$3.16 \times 10^{-3}$	95
Loud Earphones	$6.31 \times 10^{-3}$	98
Front row at concert	$1.02 \times 10^{-1}$	110

2. Intensity of 1 pupil =  $1 \times 10^{-6} \text{ Wm}^{-2}$   
 Intensity of 9 pupils =  $9 \times 10^{-6} \text{ Wm}^{-2}$   
 New dB level = 69.5 dB
3. Intensity of person talking =  $1 \times 10^{-6} \text{ Wm}^{-2}$   
 Intensity of vacuum cleaner =  $1.58 \times 10^{-5} \text{ Wm}^{-2}$   
 Total intensity =  $1.68 \times 10^{-5} \text{ Wm}^{-2}$   
 New dB level = 72.3 dB
4. a)  $1 \text{ pWm}^{-2}$ , b)  $33\,000 \text{ pWm}^{-2}$

## 9. Exponentials

In science many variations of quantities, one with respect to another, are not in a linear fashion. The change, growth or decrease, may occur rapidly. One example of such is the exponential variation with time of radioactive decay.

The characteristic of an exponential decay is that it represents a quantity which **decreases by the same fraction in successive equal time intervals**.

### Radioactive Decay

This is a relationship which will have already been encountered at GCSE.

Radioactivity is a naturally occurring phenomena which describes how the radioactive decay of a substance reduces over a period of time.

In radioactive decay, the relationship between activity  $A$  in becquerels Bq and the time  $t$  is written as:

$$A = A_0 e^{-\lambda t} \quad \text{where } A_0 = \text{activity at time } = 0$$

$$A = \text{activity at time } = t$$

$$\lambda = \text{the decay constant}$$

### Euler's number $e$

$e$  is the base rate of growth shared by continually growing processes. It is used when systems grow or decay exponentially.  $e$  is an irrational number which is equal to 2.718...

When we have a system which grows exponentially the equation looks like this:

$$y = y_0 e^{\lambda t}$$

The value of  $y$  will increase from its initial value,  $y_0$  over a period of time,  $t$  with the growth constant  $\lambda$  determining how fast the growth is over time.

When we have a system which decays (gets smaller) exponentially the equation looks like this:

$$y = y_0 e^{-\lambda t}$$

The value of  $y$  will decrease from its initial value,  $y_0$  over a period of time,  $t$  with the decay constant  $\lambda$  determining how fast the decay is over time.

### The decay constant $\lambda$

The decay constant  $\lambda$  for a radioactive sample is defined as the probability per unit time that a given nucleus of that nuclide will decay.

It is the fraction of nuclei in the sample that will decay in a unit of time.

The decay constant which is related to the half-life of the radioactive source,  $t_{1/2}$  as follows:

$$t_{1/2} = \frac{0.693}{\lambda}$$

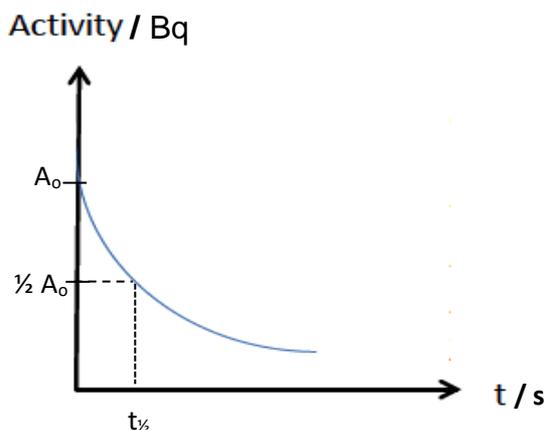
E.g. If the decay constant  $\lambda = 0.067 \text{ min}^{-1}$  then that means the half-life is 10.3 minutes (the activity will half every 10.3 minutes). The same decay constant can be expressed as  $\lambda = 0.00112 \text{ s}^{-1}$  and the half-life is 618 seconds. The decay constant can be expressed in units of  $\text{s}^{-1}$ ,  $\text{min}^{-1}$ ,  $\text{hr}^{-1}$ ,  $\text{day}^{-1}$ ,  $\text{yr}^{-1}$  which ever is most useful for the question.

### Graphical relationships

The graph below shows the relationship between activity  $A$  and time  $t$ .

The equation  $A = A_0 e^{-\lambda t}$  represented by the graph below shows a decay curve which has an initial activity  $A_0$  at time  $t = 0$ .

The value of the activity decreases over time but is never zero.



The initial activity  $A_0$  can be found by reading the Y intercept.

The half-life can be found by finding the value of the Y axis where the activity is half of its original value and reading the corresponding value of time.

The equation  $A = A_0 e^{-\lambda t}$  can be rearranged to provide an equation that will allow  $\lambda$  to be calculated. This is done by taking  $\log_e$  (or  $\ln$ ) of each side of the equation.

Using our previous log rules:

$$\ln A = \ln (A_0 e^{-\lambda t})$$

$$\ln A = \ln A_0 + \ln e^{-\lambda t}$$

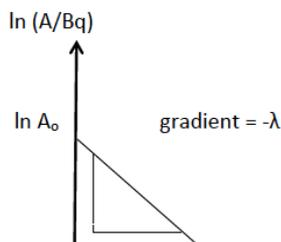
$$\ln A = \ln A_0 - \lambda t$$

this is written as

$$\ln A = -\lambda t + \ln A_0$$

$$\begin{array}{ccc} \downarrow & \downarrow \downarrow & \downarrow \\ y & = m x + c \end{array}$$

A graph of  $\ln A$  against  $t$  will give a straight-line graph with gradient =  $-\lambda$  and a positive Y intercept of  $\ln A_0$ . To find  $A_0$  from this graph, first find the Y intercept and then use the inverse  $\ln$  function on the calculator  $e^x$ .



$e^{(Y \text{ intercept})} = \text{initial activity}$

To find  $\lambda$  from this graph, find the magnitude of the gradient.

To find  $t_{1/2}$  from this graph, first calculate  $\lambda$ , then substitute this into the half-life formula.

### Practice Questions

1. Plot the graph of Activity against time and use it to determine the half-life of this radioactive material?

Time /s	0	20	40	60	80	100	120	140	160	180
Activity /Bq	495	390	307	242	190	150	118	93	73	58

2. Use the following data to plot a  $\ln A$  against  $t$  graph and use it to find the decay constant  $\lambda$  and the half-life from the gradient of the graph.

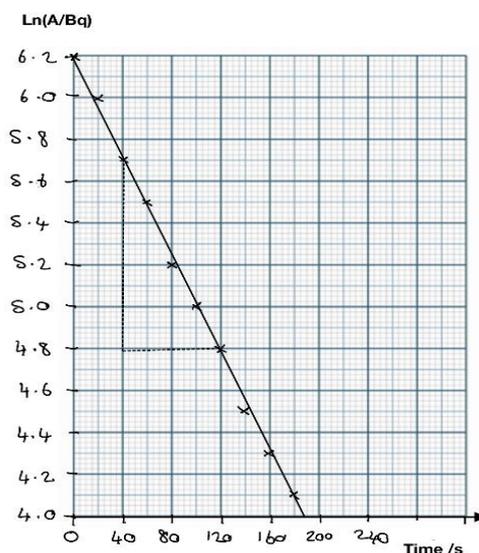
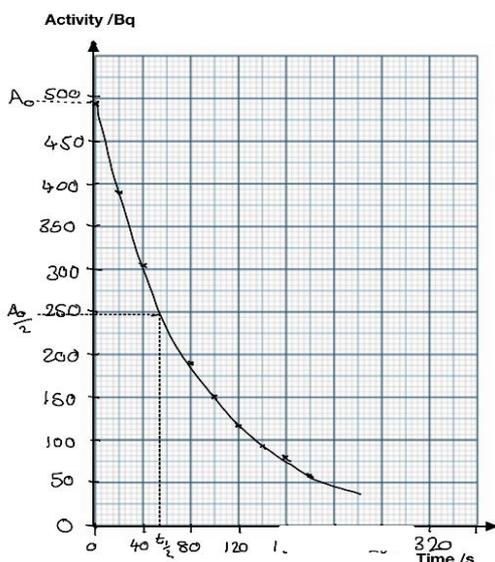
Time /s	0	20	40	60	80	100	120	140	160	180
Activity /Bq	495	390	307	242	190	150	118	93	73	58

### Answers

1.  
From the graph, the half-life is 55 s

2.

T/s	0	20	40	60	80	100	120	140	160	180
A /Bq	6.2	6.0	5.7	5.5	5.2	5.0	4.8	4.5	4.3	4.1



## Practice Questions

Find the decay constant for each of the elements in the table below:

1.

Element	Half-life $t_{1/2}$	Decay constant $\lambda$
Americium-241	432 years	
Sodium-24	15 hours	
Polonium-216	0.145 seconds	

2. The half-life of radium-226 is 1622 years.

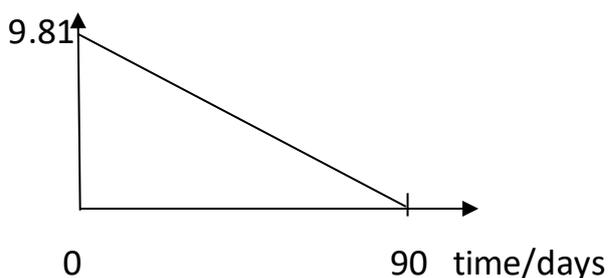
(a) Find the decay constant.

(b) Find the activity after 5000 years if the initial activity of the sample is 85560 Bq.

(c) By what percentage has the activity reduced?

3. Use the following graph to find a) the initial activity, b) the decay constant and c) the half-life of the sample.

$\ln(A/\text{Bq})$



## Answers

1.

Element	Half-life $t_{1/2}$	Decay constant $\lambda$
Americium-241	432 years	$1.60 \times 10^{-3} \text{ yrs}^{-1}$
Sodium-24	15 hours	$0.0462 \text{ hrs}^{-1}$
Polonium-216	0.145 seconds	$4.78 \text{ s}^{-1}$

2. a)  $4.27 \times 10^{-4} \text{ yr}^{-1}$ ; b) 10100 Bq; c) 88.2%

3. a) 18.2 Bq; b)  $\lambda = -\text{gradient} = 0.109 \text{ day}^{-1}$ ; c) 6.36 days

