GCSE

Rewarding Learning

CCEA GCSE
Exemplifying Examination Performance

Further Mathematics

This is an exemplification of candidates' performance in GCSE examinations (Summer 2019) to support the teaching and learning of the Further Mathematics specification.

EXEMPLIFYING EXAMINATION PERFORMANCE

GCSE Further Mathematics

Introduction

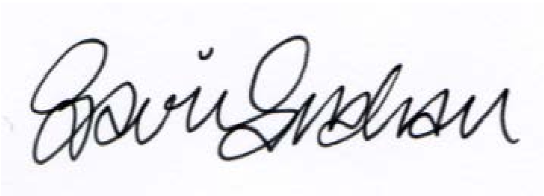
These materials illustrate aspects of performance from the 2019 summer GCSE examination series of CCEA's revised GCSE Specification in 2017.

Students' grade A responses are reproduced verbatim and accompanied by commentaries written by senior examiners. The commentaries draw attention to the strengths of the students' responses and indicate, where appropriate, deficiencies and how improvements could be made.

It is intended that the materials should provide a benchmark of candidate performance and help teachers and students to raise standards.

For further details of our support package, please visit our website at www.ccea.org.uk

Best wishes



Gavin Graham

Education Manager, Further Mathematics

Email: ggraham@ccea.org.uk

Telephone: 028 9026 1200   ext. 2658

GCSE: Further Mathematics:

Unit 1: Pure Mathematics

Grade: A Exemplar

Q1 Matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$$

Find the value of $\mathbf{B} - \mathbf{A}^2$ [4]

Student's response

$$\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} (2\times 2)+(3\times -1) & (2\times 3)+(3\times 4) \\ (-1\times 2)+(4\times -1) & (-1\times 3)+(4\times 4) \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 18 \\ -6 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -20 \\ 3 & -8 \end{bmatrix}$$

Answer $\begin{pmatrix} 0 & -20 \\ 3 & -8 \end{pmatrix}$

Examiner's comments

Q1 4/4

The candidate correctly substitutes both matrices into the expression

$$\mathbf{B} - \mathbf{A}^2$$

They correctly evaluate $\mathbf{A}^2$ including the detail of the calculation, giving the candidate the ability to check visually that they have performed the operation correctly. Finally, the two terms are subtracted to yield the correct result. This answer includes an admirable level of detail.

The question would have been adequately answered if the candidate had begun by merely evaluating $\mathbf{A}^2$ then subtracting the result from matrix **B.** This would have obtained full marks, although the detail of the development would have been omitted, making it harder for the candidate to check their work.

Q2 A function f (x) is defined by

$$f(x) = x^2 - x + 4$$

Q2(i) Use the method of **completing the square** to rewrite f (x) in the form

$$(x + a)^2 + b$$

where a and b are constants. [2]

Student's response

$x^2 - x + 4 \equiv (x+a)^2 + b$

$= x^2 + 2ax + a^2 + b$

$2ax = -x$

$2a = -1$

$a = -\frac{1}{2}$

$a^2 + b = 4$

$(-\frac{1}{2})^2 + b = 4$

$-\frac{1}{4} + b = 4$

$b = 4.25$

Answer $(x - ½)^2 + 4¼$

Examiner's comments

Q2(i) 1/2

The candidate commences by identifying the given quadratic with the desired form developing the right-hand side. Then the candidate correctly identifies the x terms and the constant terms separately and correctly. This approach includes an excellent level of detail.

A mark is lost because the candidate incorrectly evaluates $\left(-\frac{1}{2}\right)^2$ to be $-\frac{1}{4}$ so the value for b is incorrect.

Q2(ii) **Hence** find the minimum value of f (x) and the value of x for which it occurs.

Student's response

Answer Minimum value *4 ¼* [1]

when x = ½ [1]

Examiner's comments

Q2(ii) 2/2

Without explaining why, the candidate assumes (correctly) that the minimum occurs when the bracket

$$\left(x-\frac{1}{2}\right)^2$$

is equal to zero. The candidate's minimum value 4.25, although wrong, is given a mark as it follows on from their expression in (i). The value of x that makes the bracket zero is correct as 0.5

Q3(a) Find $\frac{dy}{dx}$ if $y = \frac{3}{8}x^2 + \frac{5}{x^4} - 12x$ [3]

Student's response

$$y = \frac{3x^2}{8} + \frac{5x^{-4}}{1} - 12x$$

$$\frac{dy}{dx} = \frac{6x}{8} + -20x^{-5} - 12$$

$$= \frac{3x}{4} - \frac{20}{x^5} - 12$$

Answer $\frac{3x}{4} - \frac{20}{x^5} - 12$

Examiner's comments

Q3(a) 3/3

The candidate correctly begins by replacing $\frac{5}{x^4}$ with $5x^{-4}$

This is correctly differentiated term by term without simplifying. On the last line the candidate simplifies fully and replaces x^{-5} with $\frac{1}{x^5}$

Q3(b) Find $\displaystyle\int\left(\frac{3}{4x^2} - 2x^3\right)\mathrm{d}x$ [3]

Student's response

$$\int\left(\frac{3x^{-2}}{4} - 2x^3\right)dx$$

$$= \frac{3x^{-1}}{4x-1} - \frac{-2x^4}{4}$$

$$= \frac{3x^{-1}}{-4} + \frac{1}{2}x^4$$

$$= -\frac{3}{4x} + \frac{1}{2}x^4$$

Answer $-\dfrac{3}{4x} + \dfrac{1}{2x^4}$

Examiner's comments

Q3(b) 1/3

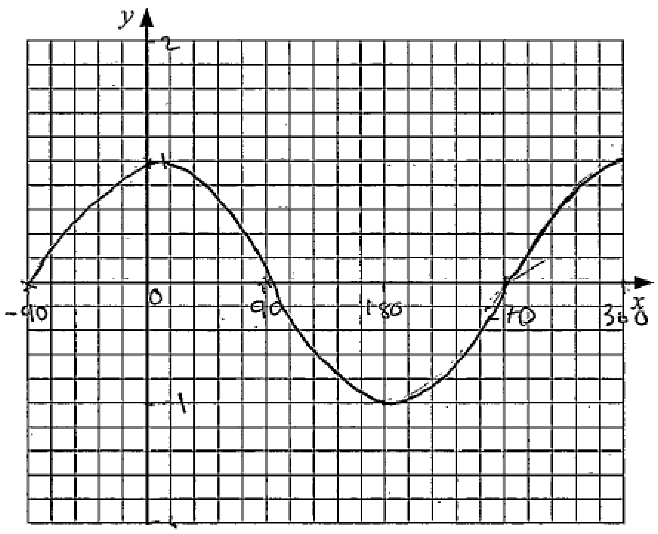
The candidate correctly begins by replacing $\frac{3}{4x^2}$ with $\frac{3}{4}x^{-2}$

This is correctly integrated, term by term, without simplifying. On the last line the candidate simplifies fully and replaces x^{-1} with $\frac{1}{x}$

The candidate loses the easiest mark on the paper forgetting the constant of integration. If the candidate had omitted the answer line, they would have obtained 2 marks for the part. However, the last term is transcribed to the answer line incorrectly, losing a mark.

Q4(a) Sketch the graph of $y = \cos x$ for $-90° \leq x \leq 360°$ [2]

Student's response



Examiner's comments

Q4(a) 2/2

The candidate carefully sketches cos(x) on the given domain, being careful to identify the correct intercepts of the x-axis and drawing the turning points in a careful and rounded fashion.

Q4(b)(i) Solve the equation

$$\cos x = -0.184$$

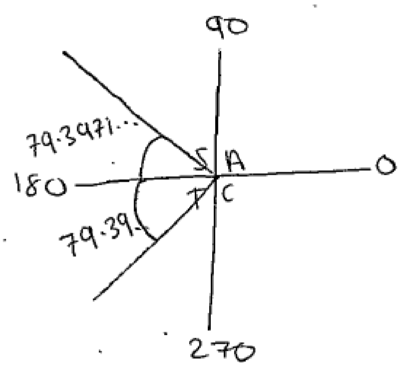
for $0° \leq x \leq 360°$

Give your answers correct to 1 decimal place. [2]

Student's response

$x = \cos^{-1}(0.184)$

$\therefore x = 79.39716....$



$x = 100.6$

$x = 259.4$

Answers $x = 100.6°$ $x = 259.4°$

Examiner's comments

Q4(b)(i) 2/2

In solving the trigonometric equation to find the principal solution, the candidate is careful to use the correct notation for the inverse cosine function. Then a careful CAST diagram is drawn identifying the correct quadrants and consequently the correct angles. The values are given to 1 decimal place as requested.

Q4(b)(ii) **Hence** solve the equation

$$\cos(2\theta - 15°) = -0.184$$

for $90° \leq \theta \leq 180°$

Give your answer correct to 1 decimal place. [2]

Student's response

$(2\theta - 15) = \cos^{-1}(-0.184)$

$2\theta - 15 = 100.6$

$2\theta = 115.6$

$\theta = 57.8$

$(2\theta - 15) = \cos^{-1}(-0.184)$

$2\theta - 15 = 259.4$

$\theta = 137.2$

Answer θ = *137.2°*

Examiner's comments

Q4(b)(ii) 2/2

The solution of this equation is developed in full detail. The two values from the first part are separately equated with $2\theta - 15^o$, and the simple equations solved correctly. The value that falls outside the given domain is discounted by a cross. The correct value is reported to 1 decimal place as requested.

Q5 Matrices **P** and **Q** are defined by

$$\mathbf{P} = \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix} \text{ and } \mathbf{Q} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

Using a matrix method, find the matrix **X** such that

$$\mathbf{PX} = \mathbf{Q}$$ [4]

Student's response

$$\begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

$$\frac{1}{10}\begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix} X = \frac{1}{10}\begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

$$X = \frac{1}{10}\underset{2\times 2}{\begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix}}\underset{2\times 1}{\begin{bmatrix} 7 \\ 1 \end{bmatrix}}$$

$$X = \frac{1}{10}\begin{bmatrix} \cancel{(-4\times 7)+(-2\times 7)} \\ \cancel{(-1\times -3)} \end{bmatrix}\begin{bmatrix} \cancel{-42} \ -30 \\ -10 \end{bmatrix}$$

$$X = \frac{1}{10}\begin{bmatrix} -30 \\ -10 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

Answer $x = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

Examiner's comments

Q5 4/4

The candidate rewrites the matrix equation substituting the explicit matrices for their capital letter equivalents. On the second line, the inverse of the matrix is calculated and multiplied on the left hand side to both sides of the matrix equation. (A slip on the left hand side in line 2 is corrected on the next line.) The candidate then calculates $\mathbf{P}^{-1}\mathbf{Q}$ using matrix multiplication to find **X.**

Q6 Solve the inequality

$$x^2 - 4x - 5 < 0$$

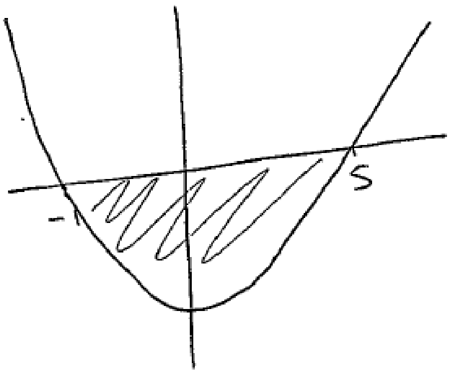
You **must** show clearly each stage of your solution. [4]

Student's response

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5 \quad x = -1$



$-1 < x < 5$

Answer $-1 < x < 5$

Examiner's comments

Q6 4/4

The candidate approaches this question using the easiest possible method. First the quadratic inequality is solved as a quadratic equality to find its 2 roots. Then the curve is sketched roughly, showing the shape of a positive quadratic curve crossing the x-axis at the calculated values. As the expression is required to be negative, the portion of the x-axis for which the curve has negative values is reported as an inequality $-1 < x < 5$

Q7 Solve the following set of simultaneous equations

$$2x + 3y + \ z = 5$$
$$3x - 4y + 2z = -9$$
$$x + 5y - 3z = 6$$

You **must** show clearly each stage of your solution. [8]

Student's response

$2x+3y+z=5$ (1)
$3x-4y+2z=-9$ (2)
$x+5y-3z=6$ (3)

(1) × 3 $\quad 6x+9y+3z=15$
(3) (+) $\quad x+5y-3z=6$
$7x+14y=21$
$x+2y=3$ (4)

(1) × 2 $\quad 4x+6y+2z=10$
(2) (−) $\quad 3x-4y+2z=-9$
$x+10y=19$ (5)

$x+10y=19$ (4)
(−) $x+2y=3$ (5)
$8y=16$
$y=2$

$x+2y=3$
$x+4=3$
$x=-1$

$3(-1)-4(2)+2z=-9$
$-3-8+2z=-9$
$2z=2$
$z=1$

Answer $\quad x =$ __*-1*__, $y =$ __*2*__, $z =$ __*1*__

Examiner's comments

Q7 8/8

The candidate uses a successful method to solve 3 equations in 3 unknowns. Firstly the 3 equations are labelled for reference: (1), (2) and (3). Then a variable is chosen to be eliminated, in this case *z*, although any variable could be chosen at this point. The method of elimination is then employed twice using different mixes of the original equations. This results in two equations in only *x* and *y*. These are labelled (4) and (5). This pair of 2 by 2 simultaneous equations is then solved via the usual approach. This yields the values of *x* and *y*. The value of *z* is then calculated by substituting *x* and *y* into one of the original equations, in this case (2). It is noted that this approach will work with any 3 by 3 set of equations at this level.

Q8 A curve is defined by the equation $y = x(3x - 5)(x + 1)$

Q8(i) Write down the **coordinates** of the points where the curve meets the x-axis. [3]

Student's response

x axis at $y=0$

$3x-5=0$

$3x=5$

$x=\frac{5}{3}$

Answer $x = 0$ $x = \frac{5}{3}$ $x = -1$

Examiner's comments

Q8 The curve sketching question – in this case a cubic through the origin.

Q8(i) 2/3

The candidate merely has to find the values of x where $y = 0$. This is straightforward as the equation is given in its factorised form. The candidate correctly finds the 3 values of x that make $y = 0$. A mark is lost as the candidate does not report these as points, rather just as x values. The correct answer would have been $(0, 0), \left(\frac{5}{3}, 0\right), (-1, 0)$.

Q8(ii) Find the coordinates of the turning points of the curve. [6]

Student's response

$$y = x(3x^2 + 3x - 5x - 5)$$
$$y = x(3x^2 - 2x - 5)$$
$$y = 3x^3 - 2x^2 - 5x$$

$$\frac{dy}{dx} = 9x^2 - 4x - 5$$

at TPs $\frac{dy}{dx} = 0$

$$9x^2 - 4x - 5 = 0$$
$$9x^2 - 9x + 5x - 5 = 0$$
$$9x(x-1) + 5(x-1) = 0$$
$$(9x+5)(x-1) = 0$$

$x = 1$ $\qquad 9x + 5 = 0$, $9x = -5$, $x = -\frac{5}{9}$

when $x = 1$, $y = -4$

when $x = -\frac{5}{9}$, $y = 1.646$

Answer $(1, -4)\ (-\frac{5}{9}, 1.646)$

Examiner's comments

Q8(ii) 6/6

The candidate starts by multiplying out the cubic equation to yield a single expression. This is then differentiated term by term correctly to obtain $\frac{dy}{dx}$. The derivative is set equal to zero to obtain a quadratic equation for the *x* values of the turning points. This quadratic equation is solved to determine the *x* values. The corresponding *y* values are found by substitution. The turning points are reported as 2 coordinate pairs. No deduction is made on this occasion although the *y* value 1.646 is not rounded to 2 decimal places as requested.

Q8(iii) Using calculus, identify each turning point as either a maximum or a minimum point. You **must** show working to justify your answer. [2]

Student's response

$$\frac{d^2y}{dx^2} = 18x - 4$$

When $x = 1$

$18(1) - 4$
$= 14 \rightarrow +ve \therefore$ minimum

$18\left(\frac{-5}{9}\right) - 4$
$= -14 \rightarrow -ve \therefore$ max

Answer $(1, -4) \longrightarrow$ *minimum*

$\left(-\frac{5}{9}, 1.65\right) \longrightarrow$ *max*

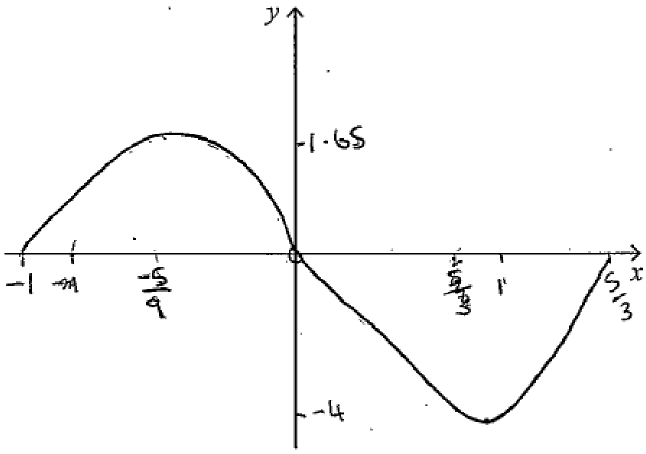
Examiner's comments

Q8(iii) 2/2

The candidate differentiates $\frac{dy}{dx}$ to obtain $\frac{d^2y}{dx^2}$. The expression for $\frac{d^2y}{dx^2}$ is then evaluated for the x values of the turning points. A negative result indicates a maximum point and a positive result indicates a minimum point.

Q8(iv) Sketch the curve on the axes below. [2]

Student's response



Examiner's comments

Q8(iv) 2/2

The candidate sketches a portion of a positive cubic equation. The 3 intercepts of the *x*-axis are labelled. The co-ordinates of the 2 turning points are labelled on the axes, which is completely sufficient. It would also have been acceptable for the co-ordinate points to have been written beside the turning points themselves.

Q9a If $2 \log y = 3 \log x$ write y in terms of x. [2]

Student's response

$2 \log y = 3 \log x$

$y^2 = x^3$

Answer $y^2 = x^3$

Examiner's comments

Q9(a) 1/2

The candidate writes out the starting equation. Then the third log rule ($n \log(a) = \log a^n$) is applied to each side. The line $\log(y^2) = \log(x^3)$ is implied. As the candidate was asked for an expression for y (= $x^{\frac{3}{2}}$) this is not a full answer, losing a mark.

Q9b(i) If $4 \times 2^x = 2^y$ show that

$$y = x + 2 \qquad [1]$$

Student's response

$a^x = n$

$x = \log_a n$

$\cancel{2^n =}$

$2^2 = 4$

$2 = \log_2 4$

$\log_2 4 \times x\log 2 = y\log 2$

$\dfrac{4 \times x\log 2}{2} = \dfrac{y\log 2}{2}$

$\cancel{2x\log 2}$

$2 \,|\, 4$
$2 \,|\, 2$
1

$4 = 2^2$

$2 + x = y$

$y = x + 2$

Examiner's comments

Q9(b)(i) 0/1

Although the candidate has written the correct answer on the page at one point, the rest of the derivation is completely incorrect, so no mark awarded.

Q9b(ii) **Hence** or otherwise solve the equation

$$6^{3x-2} = 4 \times 2^x \qquad [4]$$

Student's response

$(3x-2)\log 6 = \log 4 \times x \log 2$

$3x \log 6 - 2\log 6 = (\log 4)(x \log 2)$

Answer **No answer inserted by candidate**

Examiner's comments

Q9(b)(ii) 1/4

A correct answer for this should have started by replacing the right-hand side by 2^{x+2}, using the result from the previous part. [Note the fact that this is a part (ii) usually means it is connected in some way with the previous part (i)]. The candidate attempts to take logs of each side and then use the third log rule. As this is only correctly achieved in the left-hand side, only one mark is given.

Q10 Rory has a set of objects, each with a circular base. He records the base radius, r cm, and the volume, V cm^3, of 5 of these objects.

The results are given in the table below.

Base radius r (cm)	**Volume** V (cm³)	*log r*	*log V*
3.4	98.0	*0.531*	*1.991*
4.6	221.7	*0.663*	*2.346*
5.2	308.7	*0.716*	*2.490*
6.8	636.9	*0.833*	*2.804*
7.5	829.8	*0.875*	*2.919*

Rory believes that a relationship of the form

$$V = ar^b$$

exists, where a and b are constants.

Q10(i) Verify that a relationship of the form $V = ar^b$ exists by drawing a suitable straight line graph on the grid opposite.

Show clearly the values used, correct to 3 decimal places, in the table above.

Hence find the values of a and b, correct to 1 decimal place. [11]

Student's response

$$b = \frac{2.919 - 1.991}{0.875 - 0.531}$$

$= 2.6976\ldots$

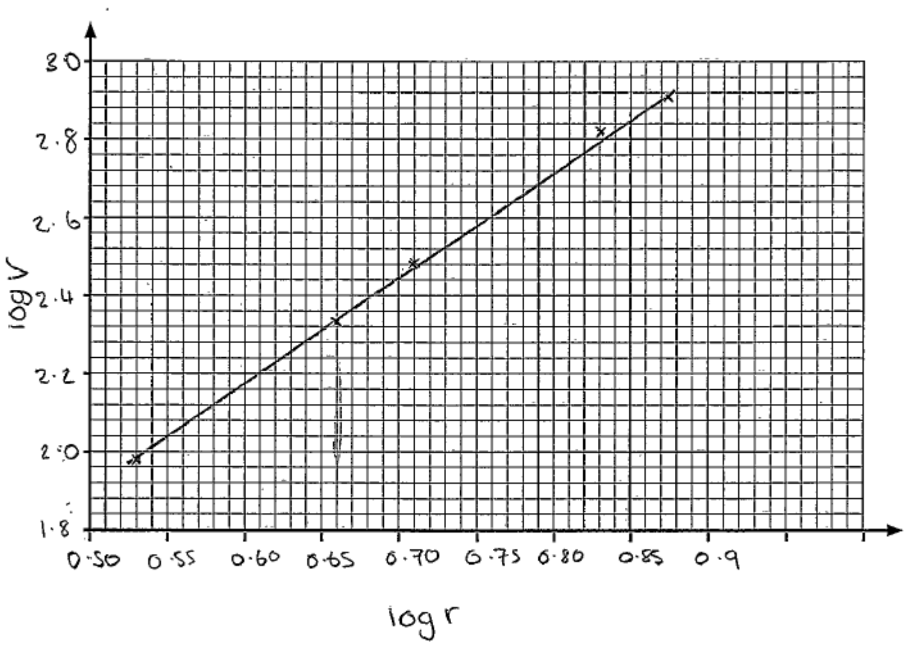
$= 2.7$

$98 = a(3.4)^{2.7}$

$98 = a(27.149\ldots)$

$a = 3.6096\ldots$

$a = 3.6$



Answer a = *3.6*, b = *2.7*

Examiner's comments

Q10(i) 10/11

The candidate correctly uses the 2 blank columns in the table, taking logs of both the given columns and writing out these values into the table correct to 3 decimal places. The relationship log(V) = b log(r) + log(a) is implied by the correctly drawn log/log graph. The axes of the graph are correctly labelled. One mark is lost in the plotting of the points. The gradient of the graph is calculated using the formula, $b = \frac{y_2 - y_1}{x_2 - x_1}$ and the value of a is calculated by substituting values in to the original equation, $V = ar^b$.

Use the formula V = ar^b with your values for a and b to calculate

Q10(ii) the volume of an object with radius 5.6 cm, giving your answer to the nearest cm^3 [1]

Student's response

$V = 3.6r^{2.7}$

$V = 3.6(5.6)^{2.7}$

$V = 377 cm^3$

Answer *377* cm^3

Examiner's comments

Q10(ii) 1/1

V is calculated by substituting r = 5.6 into the formula $V = 3.6r^{2.7}$.

Q10(iii) the **diameter** of an object with a volume of 1000 cm^3.

Give your answer correct to the nearest cm and state any assumption that you make.

Student's response

$V = 3.6r^{2.7}$

$1000 = 3.6r^{2.7}$

$r^{2.7} = \frac{1000}{3.6}$

$r^{2.7} = 277.\dot{7}$

$r = \sqrt[2.7]{277.7}$

$r = 8.0366\ldots$

$\times 2$

$d = 16.07$

Answer *16* cm [2]

Assumption *The relationship does not hold inside the given range* [1]

Examiner's comments

Q10(iii) 2/3

r is correctly calculated by substituting V = 1000 into the formula $V = 3.6r^{2.7}$. The assumption should be, "The relationship holds outside the given range".

Q11 Find the equation of the **normal** to the curve

$$y = 2 - \frac{3}{x}$$

at the point where the curve cuts the x-axis. [5]

Student's response

$y = 2 - 3x^{-1}$

$\frac{dy}{dx} = 3x^{-2}$

$= \frac{3}{x^{-2}}$

cuts x axis at $y=0$

$y = 2 - \frac{3}{x}$

$0 = 2 - \frac{3}{x}$

$-2 = -\frac{3}{x}$

$-2x = -3$

$x = \frac{2}{3}$

~~3~~

$3\left(\frac{2}{3}\right)^{-2}$

gradient of tangent $= \frac{27}{4}$

gradient of normal $= -\frac{4}{27}$

$y = mx + c$

$0 = -\frac{4}{27}\left(\frac{2}{3}\right) + c$

$c = -\frac{8}{81}$

$y = \frac{-4}{27}x - \frac{8}{81}$

Answer $y = -\frac{4}{27}x - \frac{8}{81}$

Examiner's comments

Q11 2/5

The candidate needs to find the gradient of the normal. This necessitates calculating the gradient of the tangent. This requires the derivative of the curve. $\frac{dy}{dx}$ is correctly calculated gaining a mark. The point where the curve cuts the x-axis is incorrectly calculated. The value of the gradient is incorrect. A mark is gained for the method of calculating the gradient of the normal from the tangent (by changing the sign and forming the reciprocal). The method of finding the equation of the line is correct, however this mark is not a method mark and so it isn't earned.

Q12(i) Expand and simplify the expression

$$(x + 3)(x - 4)(2x + 5) \qquad [5]$$

Student's response

$$(x+3)(2x^2 + 5x - 8x + 20)$$
$$x+3(2x^2 - 3x + 20)$$
$$x(2x^2 - 3x + 20) + 3(2x^2 - 3x + 20)$$
$$2x^3 - 3x^2 + 20x + 6x^2 - 9x + 60$$

$$\begin{array}{cccc} 2x^3 & -3x^2 & +20x & \\ & +6x^2 & -9x & +60 \\ \hline 2x^3 & +3x^2 & +11x & +60 \end{array}$$

Answer *$2x^3 + 3x^2 + 11x + 60$*

Examiner's Comments

Q12(i) 2/3

The candidate adopts a commendable strategy for multiplying out the three linear brackets. Firstly, the product of two brackets is formed. The resulting quadratic expression is tidied up by combining the two middle terms to form a three-term expression. This is then multiplied by the third linear bracket which is expanded by the distributive law. The resulting six terms are then combined. The candidate again uses a safe strategy to add the terms, listing them in columns corresponding to powers of *x*. The resulting cubic is then formed.

This layout gives the candidate a good opportunity to review their work for checking purposes.

Unfortunately, the candidate makes a slip in the very first line multiplying (−4) by (5) to give (+20). This small error is then propagated correctly through the remainder of the question.

Note the importance of seeking to consider the accuracy of each calculation in algebra. This underscores the foundational importance of practicing the solutions of quadratic problems to achieve accuracy.

Q12(ii) **Hence** simplify fully the expression [4]

$$\frac{(x+3)(x\ -4)(2x+5)-2x(x^2+2)-18}{x^2-13x}$$

Student's response

$$\frac{+3x^2+11x+60-2x^2-4x-18}{x(x-13)}$$

$$\frac{3x^2+7x+42}{x(x-13)}$$

Answer **No answer inserted by candidate**

Examiner's comments

Q12(ii) 2/4

The candidate realizes the implication of the word "hence" in the posing of the question. The mark for replacing the triple bracket by its cubic equivalent from the previous part is gained, whether the replacement is correct or, as is the case here, is not.

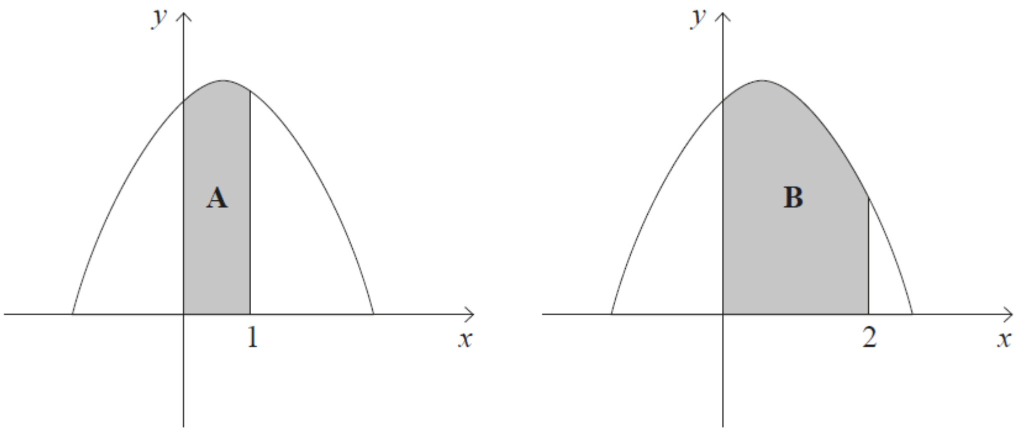
The candidate then correctly multiplies out the remaining terms in the numerator and combines them to form an overall quadratic (the terms in x^3 cancel correctly).

Unfortunately, the resulting quadratic in the numerator cannot be factored due to the mistake from the first part. This attempt ends here whereas, in the correct solution, the quadratic has a factor of (x – 13) that cancels with the same factor in the denominator.

Q13 The diagrams below show sketches of the curve

$$y = k + 2x - 3x^2$$

where k is a constant.



Q13(i) Find expressions in terms of k for the area of **A** and the area of **B**.

Student's response

$\int y = k + 2x \cancel{+} - 3x^2$

$= \frac{k^2}{2} + \frac{2}{2}x^2 - \frac{3}{3}x^3$

$= \frac{k^2}{2} + x^2 - x^3$

$\int (k + 2x - 3x^2)\,dx$

$B = \left[\frac{k^2}{2} + x^2 - x^3\right]_0^2$

$\left[\frac{k^2}{2} + (2)^2 - (2)^3\right] - \left[\frac{k^2}{2} + (0)^2 - (0)^3\right]$

$\cancel{B} \quad \frac{k^2}{2} - 4 - \frac{k^2}{2}$

-4

Area of **A** $\left[\frac{k^2}{2} + x^2 - x^3\right]_0^{21}$ [3]

Area of **B** $\left[\frac{k^2}{2} + x^2 - x^3\right]_6^2$ [2]

Examiner's comments

Q13(i) 3/5

One mark is gained by the candidate for knowing to integrate to calculate areas under the curves. Apart from that, no merit is gained in this question apart from the two answer lines. In each answer line, the incorrect result of integration is integrated between the correct *x*-values, gaining one mark for each.

Q13(ii) Given that the area of **A** is $\frac{5}{9}$ of the area of B, find the value of *k*. [2]

Student's response

Answer **No answer inserted by candidate**

Examiner's comments

Q13(ii) 0/2

No attempt is made.

Q14 The owners of a hotel wish to build a swimming pool in the hotel grounds. They plan to build a rectangular pool ABCD for adults, of length x m and width y m.

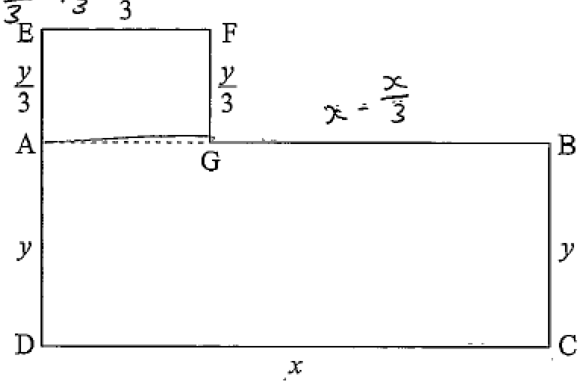
At one end of the pool they plan to add a children's pool AEFG, as shown in the diagram below.

The length and width of the children's pool are to be $\frac{1}{3}$ of the length and width of the adults' pool, respectively.

$\left(x - \frac{x}{3}\right) + y + x + y + \frac{y}{3} + \frac{x}{3} + \frac{y}{3}$

$x + 2y + (x - \cancel{\frac{x}{3}}) + \frac{y}{3} + \cancel{\frac{x}{3}} + \frac{y}{3}$

$2x + 2y + \frac{y}{3} + \frac{x}{3}$



Write down, in terms of x and y,

Q14(i) the total area of the two pools, [1]

Student's response

$\left(\frac{y}{3}\right) \times \left(\frac{x}{3}\right) + xy$

$\frac{xy}{9} + \frac{xy}{1}$

$\frac{xy + 9xy}{9} = \frac{10xy}{9}$

$\left(\frac{x}{3} \times \frac{y}{3}\right) + xy$

$\frac{xy}{9} + xy$

Answer $\frac{xy}{9} + xy$ m²

Examiner's comments

Q14(i) 1/1

The candidate correctly calculates the 2 areas of the 2 rectangles, then adds these together. The final answer $\frac{10xy}{9}$ is deleted off the answer line and replaced by the previous line. Enough is done to merit the mark.

Q14(ii) the total length of the perimeter round the outer edges of the pools. [1]

Student's response

$$\frac{x}{3}+\frac{y}{3}+\left(x-\frac{x}{3}\right)+y+x+y+\frac{y}{3}$$

$$=\cancel{\frac{x}{3}}+\frac{y}{3}+x-\cancel{\frac{x}{3}}+y+x+y+\frac{y}{3}$$

$$\frac{y}{3}+\frac{x}{1}+\frac{y}{1}+\frac{x}{1}+\frac{y}{1}+\frac{y}{3} \qquad \frac{y}{3}+\frac{3x}{3}+\frac{3y}{3}+\frac{3x}{3}+\frac{3y}{3}+\frac{y}{3}$$

$$\cancel{\frac{x}{3}}+\frac{y}{3}+\left(x-\cancel{\frac{x}{3}}\right)+y+x+y+\frac{y}{3}$$

$$\frac{y}{3}+x+y+x+y+\frac{y}{3}$$

$$\cancel{\frac{y+3x+3y}{3}}+3$$

$$\frac{y}{3}+\frac{2x}{1}+\frac{2y}{1}+\frac{y}{3}$$

$$=\frac{y}{3}+\frac{6x}{3}+\frac{6y}{3}+\frac{y}{3}\quad\times 3$$

$$=6x+8y=\text{perimetre.}$$

Answer $6x + 8y$ m

Examiner's comments

Q14(ii) 0/1

The correct lengths that sum to the perimeter are listed in the body of the answer however, the correct final answer on the line is scored out and the answer given is incorrect.

The total length of the perimeter round the outer edges of the pools is 96 m.

Q14(iii) Show that

$$y = 36 - \frac{3}{4}x \qquad [2]$$

Student's response

$6x + 8y = 96$

$8y = 96 - 6x$

~~4y = 28x~~ $\cancel{8}$ $4y = 48 - 3x$

$y = 12 - \frac{3}{4}x$

$6x + 8y = 96$

$8y = 96 - 6x$

$4y = 48 - 3x$

$y = \frac{48}{4} - \frac{3}{4}x$

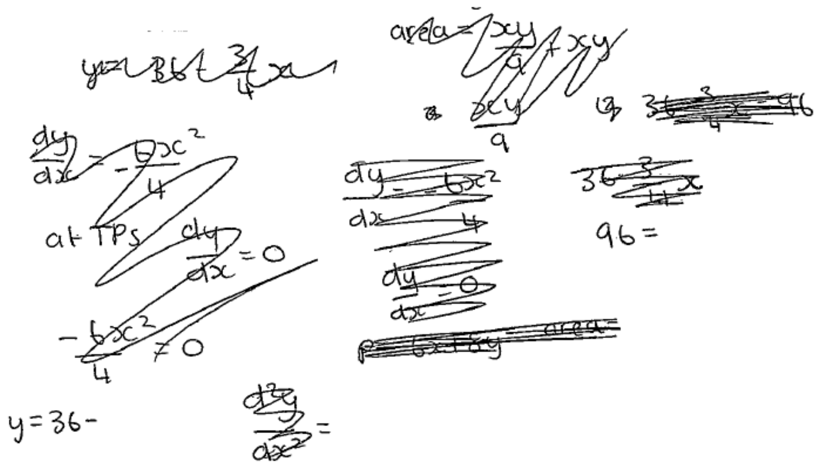
Examiner's comments

Q14(iii) 1/2

A mark is gained for equating the (wrong) expression from part (ii) with the known value of the perimeter, 96 m. Nothing else is gained.

Q14(iv) Find the dimensions of the pools which will give a maximum total area, showing that it is a maximum. [6]

Student's response



Adults' pool **No answer** *m by* **No answer** *m*

Children's pool **No answer** *m by* **No answer** *m*

Examiner's comments

14(iv) 0/6

All work is scored out. No marks.

There is nothing of further value in the Supplementary Answer Sheet.

GCSE: Further Mathematics:

Unit 2: Mechanics

Grade: A Exemplar

Q1(i) Define a vector quantity. Include an example in your answer.

Student's response

Definition *a vector quanity is a quanity that includes both magnitude and direction* [1]

Example *velocity, displacement* [1]

Q1(ii) Define a scalar quantity. Include an example in your answer.

Student's response

Definition *a scalar quanity is a quanity that includes only magnitude and has no direction* [1]

Example *speed, distance* [1]

Examiner's comments

Q1(i) and (ii) 4/4

This candidate has both the definition and examples given of vector and scalar quantities correct for full marks.

Q2 A body is initially at an origin O and is travelling with an initial velocity of $(-3\mathbf{i} + 2\mathbf{j})$ m/s.

It moves with a constant acceleration of $(4\mathbf{i} - 6\mathbf{j})$ m/s^2 for 4 seconds.

Calculate

Q2(i) the displacement of the body from O, in vector form, after the 4 seconds, [3]

Student's response

$u = -3i + 2j$

$a = 4i - 6j$

$t = 4$

$\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + 4\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$\underline{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 16 \\ -24 \end{pmatrix}$

$\underline{v} = \begin{pmatrix} 13 \\ -22 \end{pmatrix}$

$\underline{s} = \frac{1}{2}ut + at^2$

$\underline{s} = \frac{1}{2}4\begin{pmatrix} -3 \\ 2 \end{pmatrix} + 4^2\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$\underline{s} = 2\begin{pmatrix} -3 \\ 2 \end{pmatrix} + 16\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$\underline{s} = \begin{pmatrix} -6 \\ 4 \end{pmatrix} + \begin{pmatrix} 64 \\ -96 \end{pmatrix}$

$\underline{s} = \begin{pmatrix} 58 \\ -92 \end{pmatrix}$

$\underline{r} = \underline{r}_0 + \underline{v}t$

$\underline{r} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \underline{v}t$

r

$s = \frac{1}{2}ut + at^2$

$s = \frac{1}{2}\begin{pmatrix} -3 \\ 2 \end{pmatrix} \times 4 + 4^2\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$s = \begin{pmatrix} -12 \\ 8 \end{pmatrix}\frac{1}{2} + \begin{pmatrix} 64 \\ -96 \end{pmatrix}$

$s = \begin{pmatrix} -6 \\ 4 \end{pmatrix} + \begin{pmatrix} 16 \\ -24 \end{pmatrix}$

$s = 10i - 20j$

Answer *58i – 92j* m

Examiner's comments

Q2(i) 0/3

The candidate has correctly calculated the vector *v* but has also calculated the vector *s* which they have given as their final answer on the answer line. The method leading to the answer on the answer line is marked which is worth zero marks.

Q2(ii) the **speed** of the body after the 4 seconds, [4]

Student's response

speed = |velocity| $\rightarrow$ from (ii) $\rightarrow$ $13i - 22j = v$

speed = $|13i - 22j|$ $\quad$ $\underline{v} = \underline{u} + \underline{a}t$

speed = $\sqrt{13^2 + 22^2}$ $\quad$ $\underline{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + 4\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$= \sqrt{169 + 484}$ $\quad$ $\underline{v} = \begin{pmatrix} 13 \\ -22 \end{pmatrix}$

$= \sqrt{653}$

$= 25.55386468$

$= 25.55$ m/s

Answer *25.55* m/s

Examiner's comments

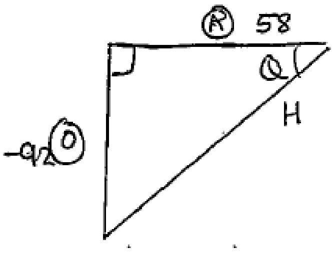
Q2(ii) 4/4

The candidate has used the correct calculation of *v* from part (i) and have correctly found the magnitude for full marks.

Q2(iii) the angle the velocity makes with the positive x-axis after the 4 seconds. [2]

Student's response

SOH CAH TOA



$$\tan Q = \frac{O}{A}$$

$$= \cancel{58}$$

$$\tan Q = \frac{-92}{58}$$

SHIFT TAN ANS

$= -57.7712$

$= 57.77$

Answer *57.77 °*

Examiner's comments

Q2(iii) 0/2

The candidate has found the angle of the calculated *s* in part (i) with the *x*-axis which is incorrect so zero marks awarded.

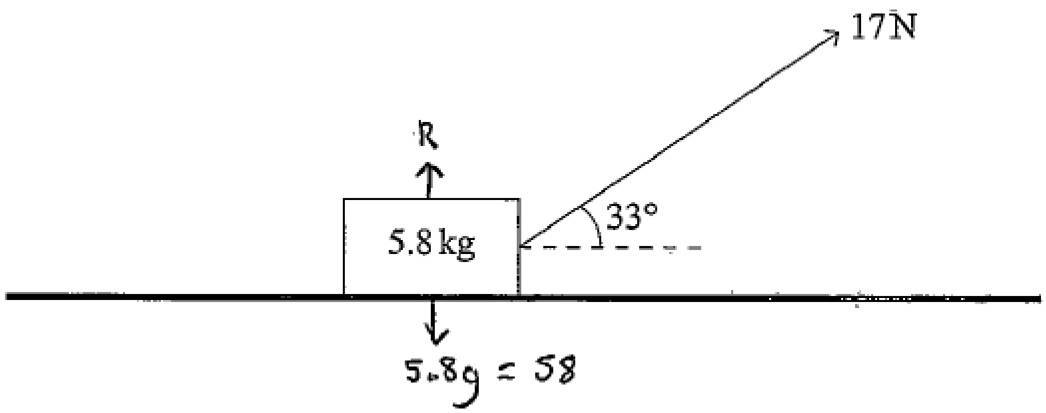
Q3 A block of mass 5.8 kg is initially at rest on a smooth horizontal table.

The block is then pulled along the table by a string with a force of 17 N. The string makes an angle of 33° to the horizontal, as shown in the diagram below.

Q3(i) Mark, on the diagram above, the other forces acting on the block. [1]

Calculate

Student's response



Examiner's comments

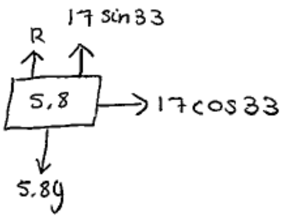
Q3(i) 1/1

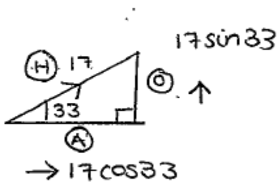
The candidate has correctly marked on the required forces for full marks. It should be noted that the majority of candidates also add on the resolved components to their diagram – this is not penalized for having additional forces added onto a diagram.

Calculate

Q3(ii) the normal reaction between the block and the table, [4]

Student's response





F = ma

vertically = equilibrium

$R + 17\sin 33 = 5.8g$

$R + 17\sin 33 = 58$

$R = 58 - 17\sin 33$

$R = 48.74113\ldots$

$R = 48.74$ to 2dp

Answer *48.74* N

Examiner's comments

Q3(ii) 4/4

The candidate has correctly resolved their forces and correctly equated vertically to gain full marks in finding the normal reaction.

Q3(iii) the acceleration of the block, [3]

Student's response

F = ma

helpful F − unhelpful F = m × a

$17\cos 33 = 5.8 \times a$

$14.25739466 = 5.8 \times a$

$\frac{14.2573\ldots}{5.8} = a$

$a = 2.458172334$

$a = 2.46$ to 2dp

Answer *2.46* m/s^2

Examiner's comments

Q3(iii) 3/3

The candidate has correctly applied Newton's Second Law horizontally to the block to gain full marks for the acceleration.

Q3(iv) the speed *v* of the block after 5 seconds. [1]

Student's response

$t = 5$

$v = ?$
$u = 0 \text{ m/s}$
$t = 5$
$a = 2.46$

$v = u + at$
$v = 0 + (2.46 \times 5)$
$v = 12.3$

however, $a \rightarrow$ unrounded

$= \frac{14.25739966}{5.8} \times 5$

$= 2.458172354 \times 5$

$= 12.29086$

$= 12.29$

12.29086
12.3

Answer *12.29* m/s

Examiner's comments

Q3(iv) 1/1

The candidate has correctly applied the correct equation of motion with the correct values to fine the required speed.

Q4 A uniform rod AB has length 3.2 m and mass 4.6 kg.

It is suspended from a ceiling by two inextensible strings attached to points C and D on the rod.

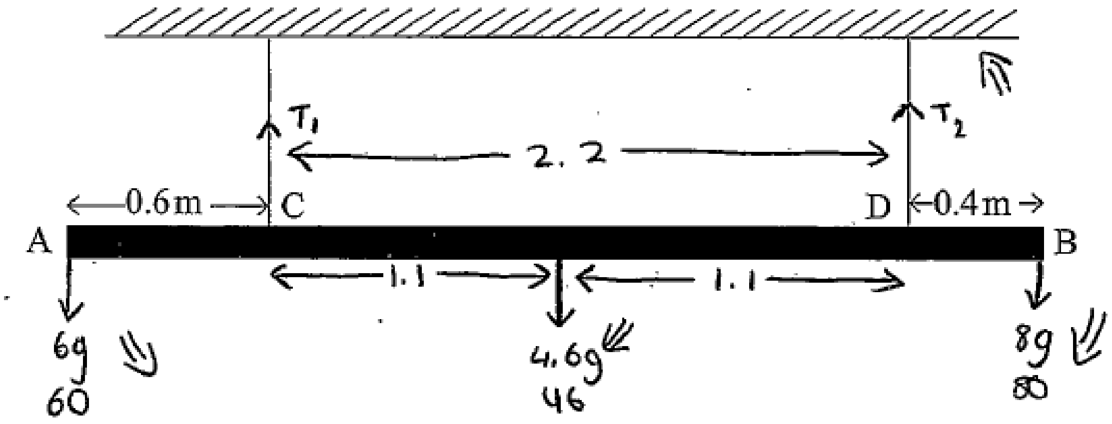
The distance AC is 0.6 m and the distance DB is 0.4 m, as shown in the diagram below.

A mass of 6 kg is attached to the rod at the end A and a mass of 8 kg is attached to the rod at the end B.

The rod remains horizontal and in equilibrium.

Q4(i) Mark, on the diagram above, all the forces acting on the rod. [2]

Student's response



Examiner's comments

Q4(i) 2/2

The candidate has correctly marked on all the forces acting on the rod. Care had to be taken when marking on the weight of the rod to ensure it is marked in the centre of the rod and not in the middle of the two strings.

Q4(ii) Calculate the tensions in the strings at C and D. [6]

Student's response

ACW = CW

take moments about C (so there is no T_1)

$(T_2 \times 2.2) + (6g \times 0.6) = (1.1 \times 4.6g) + (2.6 \times 8g)$

$(T_2 \times 2.2) + (60 \times 0.6) = (1.1 \times 46) + (2.6 \times 80)$

$2.2 \times T_2 + 36 = 50.6 + 208$

$2.2T_2 + 36 = 258.6$

$-36 \quad -36$

$2.2T_2 = 222.6$

$\div 2.2 \quad \div 2.2$

$T_2 = 101.1818182$

$T_2 = 101.1818$ ①

$T_2 = 101.18$

$T_1 + T_2 = 6g + 8g + 4.6g$

$T_1 + T_2 = 60 + 80 + 46$

$T_1 + T_2 = 186$ ②

put ① into ②

$T_1 + T_2 = 186$

$T_1 + 101.1818 = 186$

$T_1 = 84.8182$

$T_1 = 84.82$

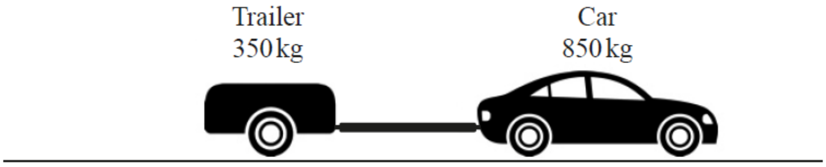
Answer	Tension in string at C	*84.82* N
	Tension in string at D	*101.18* N

Examiner's comments

Q4(iii) 3/6

The candidate has attempted to set up a moments equation with all terms being a force times a distance and at least one of the terms is an unknown for a mark. The anticlockwise moments about C are correct for a mark but there is an error in distance the weight of the rod is from C in the clockwise moments. The candidate has correctly equated vertically for a third mark.

Q5 A car of mass 850 kg tows a trailer of mass 350 kg along a straight horizontal road. The car and trailer are connected by a light horizontal towbar.



The resistance to motion of the car is 1.2 N per kg.

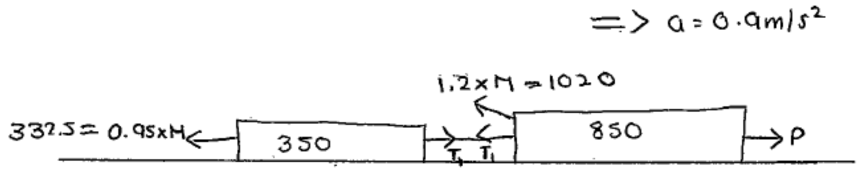
The resistance to motion of the trailer is 0.95 N per kg.

The car and trailer travel at a constant acceleration of 0.9 m/s^2

Calculate

Q5(i) the tension in the towbar, [3]

Student's response



$0.95 \times 350 = 332.5$

$1.2 \times 850 = 1020$

Use trailer

$F = ma$

helpful F − unhelpful $F = m \times a$

$T_1 - 332.5 = 350 \times 0.9$

$T_1 - 332.5 = 315$

$+332.5 \quad +332.5$

$T_1 = 647.5$

Answer *647.5* N

Examiner's comments

Q5(i) 3/3

The candidate has correctly found the resistance to motion of the trailer and has correctly applied Newton's Second Law to the trailer to correctly find the tension of the tow-bar.

Q5(ii) the tractive force of the engine of the car. [3]

Student's response

=P

overall system

helpful F - unhelpful F = m × a

$P \; \underbrace{+ \; T_1} \; \underbrace{- \; T_1} - 332.5 - 1020 = m \times a$

$P - 332.5 - 1020 = 850 \times 0.9$

$P - 1352.5 = \cancel{850}^{1200} \times 0.9$

~~P - 1352.5 = 765~~

~~P = 2117.5~~

$P - 1352.5 = 1080$

$P = 2432.5\text{N}$

check we car

$P - 1020 - T_1 = m \times a$

$P - 1020 - 647.5$

$P - 1667.5 = 850 \times 0.9$

$P - 1667.5 = 765$

$P = 2432.5$

Answer *2432.5* N

Examiner's comments

Q5(ii) 3/3

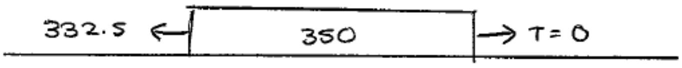
The candidate has correctly used Newton's Second Law on the car to correctly calculate the tractive force of the car.

The car and trailer started from rest.

Eight seconds later the towbar breaks.

Q5(iii) Calculate the speed of the car when the towbar breaks. [2]

Student's response



get a: $F = ma$

helpful − unhelpful $= m \times a$

$0 - 332.5 = 350 \times a$

$-332.5 = 350a$

$\div 350 \quad \div 350$

$\frac{-332.5}{350} = a$

$a = -0.95$

$v = u + at$

$v = 0 + (-0.95 \times 8)$

$v = -7.6$

$v = 7.6 \text{ m/s}$

$v = u + at$

$v = 0 + (0.9 \times 8)$

$v = 7.2$

Answer *7.2* m/s

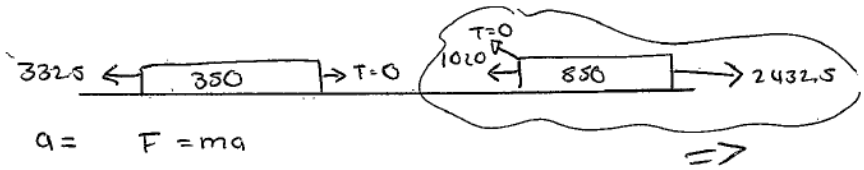
Examiner's comments

Q5(iii) 2/2

The candidate has used the correct equation of motion with the correct values substituted to gain full marks.

Q5(iv) Calculate the speed of the car 12 seconds after the towbar breaks, given that the tractive force of the car and the resistance to motion of the car remain unchanged. [5]

Student's response



a =   F = ma

helpful F − unhelpful F = m × a

2432.5 − 1020 = 850 × a

1412.5 = 850a

$\frac{1412.5}{850} = a$

a = 1.661764706

a = 1.661765

v = u + at

a = 1.661765

t = 12

u = 7.2

v = ?

v = u + at

v = 7.2 + (1.661765 × 12)

v = 7.2 + 19.94117647

v = 27.14117647

v = 27.14 to 2dp

Answer   *27.14* m/s

Examiner's comments

Q5(iv) 5/5

Again the candidate has correctly calculated the new acceleration of the car and then has gone on to correctly calculate the new velocity of the car for full marks. It should be noted that a candidate would only get follow through marks for this part of the question if an attempt of calculating a new acceleration was attempted. Also, follow through marks for the final two marks of the question to find the new velocity could only be gained if the new acceleration calculated was greater than the original 0.9 given in the question and the correct equation of motion used with *u* to equal 7.2 and *t* to equal 12.

Q6 A block of mass 7 kg lies on a rough surface inclined at an angle of 27° to the horizontal.

The force due to friction is 18.7 N.

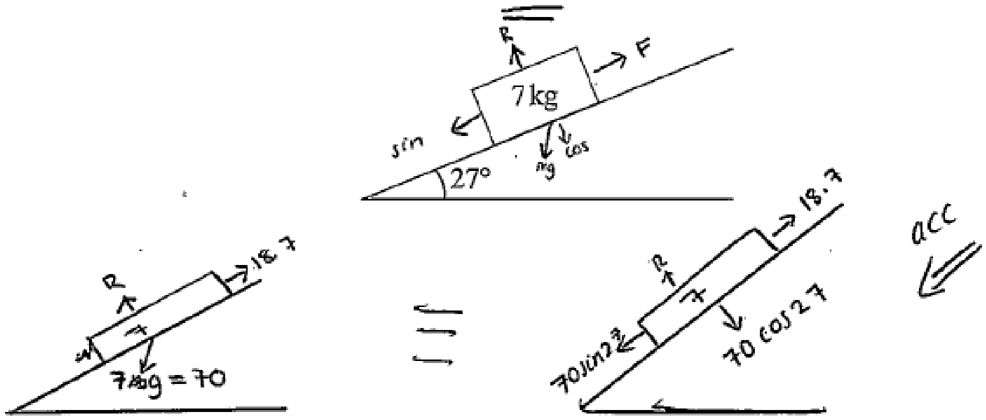
Which of the following options would produce the greatest acceleration of the block?

[7]

You must show working to support your answer.

Option A The block is allowed to slide down the slope.

Student's response



vertically equilibrium = R = 70

mgcosθ

F = m × a

helpful F − unhelpful F = m×a

70sin27 − 18.7 = 7×a

31.77933498 − 18.7 = 7a

13.07933498 = 7a

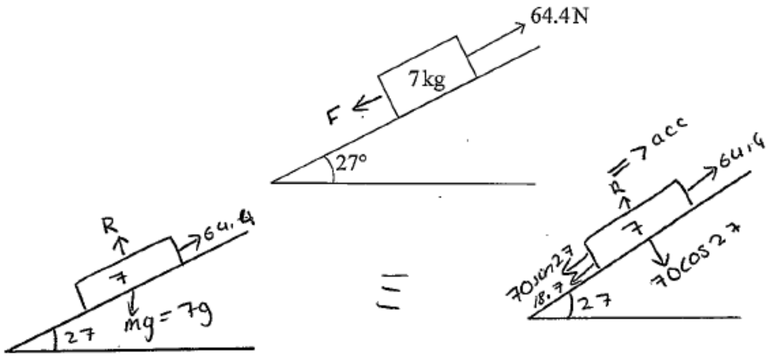
a = $\frac{13.07933498}{7}$

a = 1.868476426

a = 1.87 m/s²

Option B The block is pulled up the slope with a force of 64.4 N.

Student's response



$F = ma$

helpful F − unhelpful F = ma

$64.4 - 70\sin 27 = 7 \times a$

$64.4 - 31.77933498 = 7a$

$32.62066502 = 7a$

$\frac{32.62066502}{7} = a$

$a = 4.660095$

$a = 4.66 \text{ m/s}^2$

$64.4 - 70\sin 27 - 18.7 = 7a$

$13.92066502 = 7a$

$\div 7 \qquad \div 7$

$a = 1.98866$

$a = 1.99 \text{ m/s}^2$ to 2dp

Answer Option *B*

Examiner's comments

Q6 5/7

This question was testing Assessment Objectives 2 and 3 where the candidate had to mark on all the forces acting on the block, apply and use Newton's Second Law correctly and then make an informed decision for the final mark. In this script, the candidate has correctly found the acceleration for Option A for three marks but only gains one mark for Option B for showing evidence of a difference of forces with the 64.4 force being the greater in their calculation of 'F' in '$F=ma$'. The candidate has been awarded the final mark as one of the options had been calculated correctly and the candidate's conclusion was correct for their two calculated accelerations. The final mark for the conclusion could only be awarded if at least one of the two options had been calculated correctly and the correct resulting conclusion had been made.

GCSE: Further Mathematics:

Unit 3: Statistics

Grade: A Exemplar

Q1 A solicitor recorded the times, to the nearest minute, spent with clients. The table below shows a summary of the times.

Time (minutes)	**Frequency**	x	fx	fx^2
10-14	8	*12*	*96*	*1152*
15-19	16	*17*	*272*	*4624*
20-24	34	*22*	*748*	*16456*
25-29	27	*27*	*729*	*19683*
30-34	10	*32*	*320*	*10240*
35-39	5	*37*	*185*	*6845*

Q1(i) Calculate an estimate of the mean time. You **must** show your working. [2]

Student's response

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{2350}{100}$$

$$= \cancel{2} \; 23 \cdot 5$$

Answer *23.5* minutes

Q1(ii) Calculate an estimate of the standard deviation of the times.

You **must** show your working. [3]

Student's response

standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$

$= \sqrt{\frac{59000}{100} - (23 \cdot 5)^2}$

$= \frac{\sqrt{151}}{2}$

$= 6 \cdot 14$

Answer *6.14* minutes

Examiner's comments

Q1(i) 2/2

Q1(ii) 3/3

The candidate has accurately performed routine calculations using the formulae for mean and standard deviation of a grouped frequency distribution.

Q2 Rebekah takes 2 beads at random, without replacement, from a bag containing 6 blue and 4 green beads.

Q2(i) What is the probability that both beads are the same colour? [2]

Student's response

$$P = (BB) + (GG)$$

$$= \left(\frac{6}{10} \times \frac{5}{9}\right) + \left(\frac{4}{10} \times \frac{3}{9}\right)$$

$$= \frac{7}{15}$$

Answer $\frac{7}{15}$

Rebekah takes a third bead from the remaining 8 beads in the bag.

Q2(ii) What is the probability that all 3 beads are the same colour? [2]

Student's response

$$P = (BBB) + (GGG)$$

$$= \left(\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}\right) + \left(\frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}\right)$$

$$= 0.2$$

$$= \frac{1}{5}$$

Answer $\frac{1}{5}$

Q2(iii) Given that the first two beads are the same colour, what is the probability that the third bead is also the same colour as the first two? [2]

Student's response

$$P \not\!\!\# (A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{5} \times (7/15)}{(7/15)}$$

$$= 0.2$$

$$= \frac{1}{5}$$

Answer $\frac{1}{5}$

Examiner's comments

Q2(i) 2/2

Q2(ii) 2/2

Q2(iii) 0/2

The candidate has accurately answered 2 routine questions, (i) and (ii), involving combined probabilities. This gained 4 marks out of 6 marks.

They were unable to gain the 2 marks for a question involving conditional probability, (iii).

Q3 Two boys and twelve girls sat a piano examination.

One boy scored 95% and the other boy scored 76%.

The mean of the girls' results was 82%.

The standard deviation of the girls' results was 6%.

Q3(i) Calculate the mean of all 14 results. [2]

Student's response

$$\text{mean} = \frac{12 \times 82 + 95 + 76}{14}$$

$$= 82.5$$

Σfx^2

x	f	fx	fx^2
82	12		83640
95	1	95	9025
76	1	76	5776

Answer *82.5* %

Q3(ii) Calculate the standard deviation of all 14 results. [4]

Student's response

$$\text{standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$$

$$6 = \sqrt{\frac{12 \times x^2}{12} - (82)^2}$$

$$82^2 + 36 = \frac{12x^2}{12}$$

$$12(6760) = 12x^2$$

$$6760 = x^2$$

$$x = 26\sqrt{10}$$

$$= 82.2$$

$$\text{sd} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$$

$$= \sqrt{\frac{\cancel{\text{[illegible]}}}{14} - (82.5)^2}$$

$$\cancel{= 15.01\ \%}$$

$$= 6.52\ \%$$

Answer *6.52* %

Examiner's comments

Q3(i) 2/2

Q3(ii) 0/4

In this question the candidate was given the mean and standard deviation of twelve results. Two other results were then stated and the candidate was asked to find the mean and standard deviation of all 14 results. The candidate accurately found the mean of all 14 results, (i), using a multi-step procedure and gained 2 marks for this. However the candidate was unable to calculate the standard deviation of all 14 results, (ii), and gained none of the 4 marks for this part of the question.

Q4a Using Pascal's triangle, write out the expansion of $(p + q)^6$ [2]

Student's response

$(p + q)^6$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$= 1(p^6) + 6(p^5 \times q^1) + 15(p^4 \times q^2) + 20(p^3 \times q^3) + 15(p^2 \times q^4) + 6(p^1 \times q^5) + 1(q^6)$

$= p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$

$p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$

Answer $p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$

Q4b A bag contains a large number of red pens and black pens.

The probability that a pen, chosen at random from the bag, is black is $\frac{4}{5}$

Jill picks 6 pens, chosen at random, from the bag.

Find the probability that

Q4b(i) none of the pens is red, [3]

Student's response

$$P = 1 - \cancel{6} \left(\frac{4}{5}\right)^6$$

$$= 0.74 \text{ to } 2dp$$

Answer *0.26*

Q4b(ii) at least 2 of the pens are red. [3]

Student's response

$$P = 1 - \left(\left(\frac{4}{5}\right)^6 + \left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^5\right)$$

$$= 0.67$$

Answer *0.67*

Examiner's comments

Q4(a) 2/2

Q4(b)(i) 3/3

Q4(b)(ii) 1/3

The candidate gained 2 out of 2 marks for accurately writing out the expansion of $(p + q)^6$, using Pascal's triangle, (a). They also gained 3 out of 3 marks for using a single term of this expansion to find the probability that "none of the pens is red", (b)(i). However the candidate only gained one of the last 3 marks for finding the probability that "at least two of the pens are red", (b)(ii), because they had the correct method but their working was incorrect.

Q5 A farmer sold a large number of eggs.

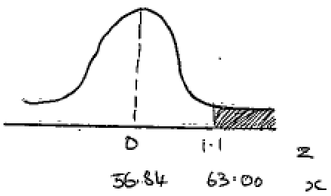
The weights of the eggs were normally distributed with mean 56.84 g and standard deviation 5.6 g.

Eggs weighing over 63 g were graded as large.

Find the probability that an egg, chosen at random, was graded as large. [4]

Student's response

$\Phi(1.1) = 0.8643$



$P(\text{Large}) = 1 - 0.8643$

$= 0.1357$

$z = \dfrac{x - \mu}{\sigma}$

$= \dfrac{63 - 56.84}{5.6}$

≈ 1.1

Answer *0.1357*

Examiner's comments

Q5 4/4

The candidate has shown their knowledge of the Normal Distribution. They have accurately calculated a *z* score and have used the Normal Probability Table to answer a complex probability question. They gained 4 out of 4 marks.

Q6 A café recorded the number of hot soups sold and the temperature outside at lunchtime on nine particular days. The results are shown in the table below.

Q6(i) Write down, in the table above, the rank orders for the temperatures and the numbers of hot soups sold. [2]

Student's response

Temperature (°C)	3	5	20	17	10	9	13	15	12
Hot soups sold	106	98	20	38	68	80	55	44	68
Ranks (Temperature)	*9*	*8*	*1*	*2*	*6*	*7*	*4*	*3*	*5*
Ranks (Hot soups sold)	*1*	*2*	*9*	*8*	*4.5*	*3*	*6*	*7*	*4.5*
d	*8*	*6*	*8*	*6*	*1.5*	*4*	*2*	*4*	*0.5*
d^2	*64*	*36*	*64*	*36*	*2.25*	*16*	*4*	*16*	*0.25*

Q6(ii) Calculate Spearman's coefficient of rank correlation. [4]

Student's response

$$r = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 238.5}{9(9^2-1)}$$

$$= -0.9875$$

Answer - *0.9875*

Q6(iii) Interpret your answer to part **(ii).** [1]

Student's response

Answer *Strong negative correlation*

Q6(iv) Calculate the mean temperature and the mean number of hot soups sold. [1]

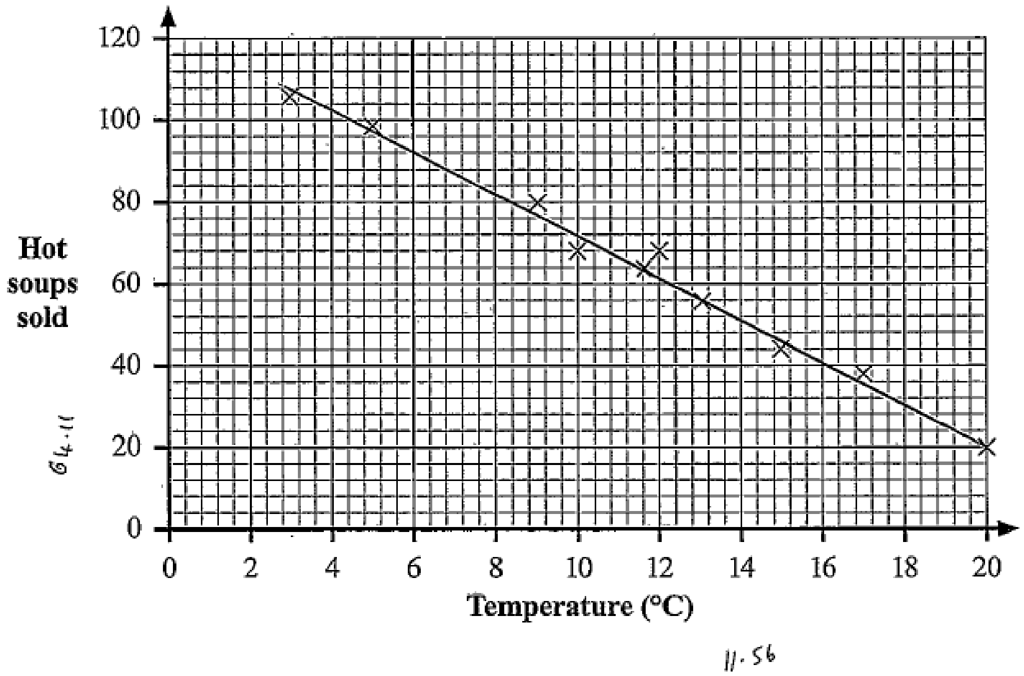
Student's response

Answer	Mean temperature	*11.56 °C*
	Mean number of hot soups sold	*64.11*

The data from the table are plotted on the graph below.

Q6(v) Draw your line of best fit on the graph. [2]

Student's response



Q6(iv) Determine the equation of the line of best fit which you have drawn. [3]

Student's response

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \cancel{\frac{14.8}{64}}$$

$$= \frac{64.1 - 92}{11.6 - 6}$$

$$= -4.98$$

$$y - y_1 = m(x - x_1)$$

$$y - 92 = -\cancel{5}_{98}(x - 6)$$

$$y = -4.98x + 121.88$$

Answer $y = -4.98x + 121.88$

Examiner's comments

Q6(i) 2/2

Q6(ii) 4/4

Q6(iii) 1/1

Q6(iv) 1/1

Q6(v) 2/2

Q6(vi) 3/3

The candidate has shown extensive knowledge of Bivariate Analysis, gaining all 13 marks in this 6 part question. They accurately calculated Spearman's coefficient of rank correlation and interpreted the result. They worked out the means, drew their line of best fit and calculated the equation of their line of best fit.

Q7 At Eastwood Boys Comprehensive, 120 new pupils were allowed to sign up for three after school sports activities – football, hockey and rugby.

Each pupil signed up for **at least** one activity.

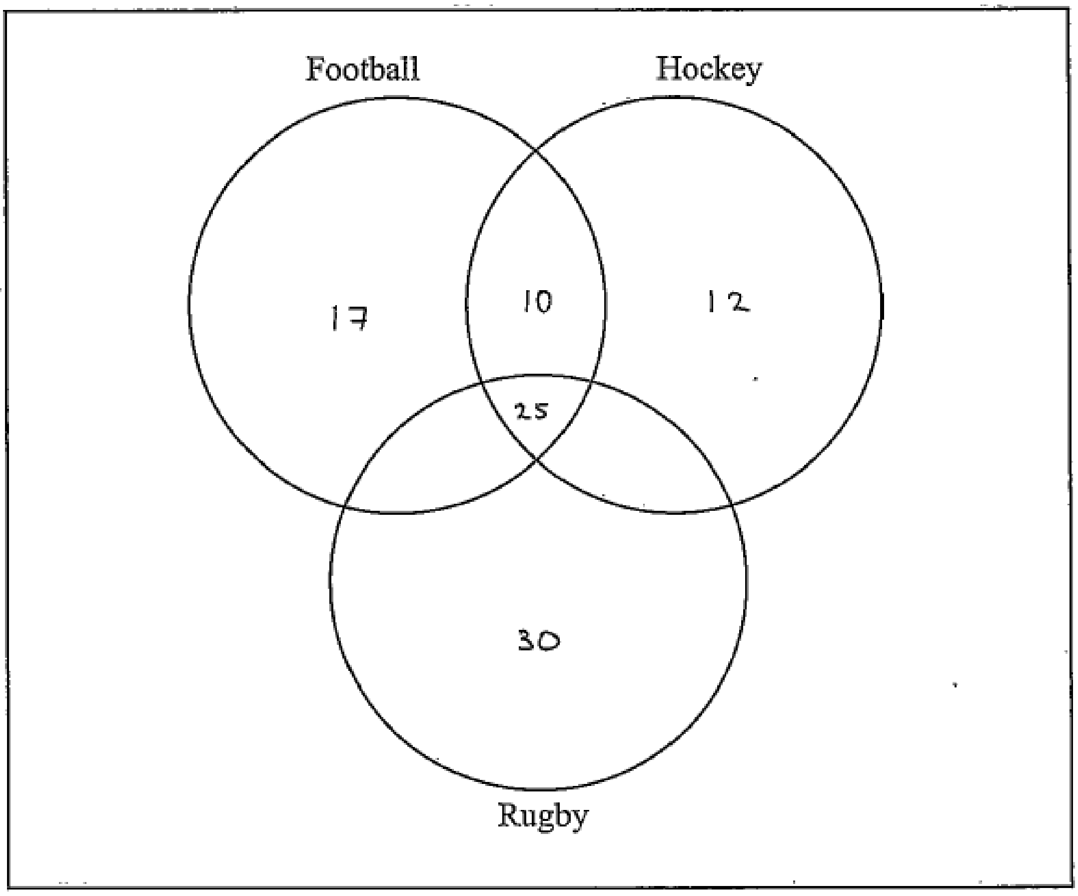
17 chose football only

12 chose hockey only

30 chose rugby only

25 chose all three activities

39 did not choose rugby



39 F + H 39

Q7(i) Using the Venn diagram opposite, find the probability that a new pupil, selected at random, chose both football and hockey. [5]

Student's response

$P = \cancel{F}\ FH + FHR$

$= \frac{10}{120} + \frac{25}{120}$

$= \frac{7}{24}$

Answer $\frac{7}{24}$

Q7(ii) Calculate the probability that a new pupil, chosen at random, chose exactly two activities. [3]

Student's response

$P = \frac{26}{120} + \frac{10}{120}$

$= \frac{36}{120}$

$= \frac{3}{10}$

Answer $\frac{3}{10}$

Examiner's comments

Q7(i) 5/5

Q7(ii) 3/3

The candidate has demonstrated their knowledge of Venn diagrams. They accurately completed a Venn diagram and used appropriate methods to solve two complex probability questions. They gained 8 out of 8 marks.

COUNCIL FOR THE **CURRICULUM, EXAMINATIONS** AND **ASSESSMENT**
29 Clarendon Road, Clarendon Dock, Belfast BT1 3BG
Tel: +44 (0)28 9026 1200 Fax: +44 (0)28 9026 1234
Email: info@ccea.org.uk Web: www.ccea.org.uk

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