GCSE

Rewarding Learning

CCEA GCSE Specification in

Further Mathematics

Version 2: 17 September 2019

For first teaching from September 2017
For first assessment in Summer 2018
For first award in Summer 2019
Subject Code: 2260

Contents

Subject Code	2330
QAN	603/1054/6

This specification is available online at www.ccea.org.uk

Version 2: Updated 17 September 2019

1 Introduction

This specification sets out the content and assessment details for our GCSE course in Further Mathematics. We have designed this specification to meet the requirements of:

- Northern Ireland GCSE Design Principles; and
- Northern Ireland GCE and GCSE Qualifications Criteria.

First teaching is from September 2017. We will make the first award based on this specification in Summer 2019.

This specification is a unitised course. The guided learning hours, as for all our GCSEs, are 120 hours.

The specification supports the aim of the Northern Ireland Curriculum to empower young people to achieve their potential and to make informed and responsible decisions throughout their lives, as well as its objectives:

- to develop the young person as an individual;
- to develop the young person as a contributor to society; and
- to develop the young person as a contributor to the economy and environment.

If there are any major changes to this specification, we will notify centres in writing. The online version of the specification will always be the most up to date; to view and download this please go to www.ccea.org.uk

1.1 Aims

This specification aims to encourage students to:

- develop further their mathematical knowledge, skills and understanding;
- select and apply mathematical techniques and methods to mathematical, everyday and real-world situations;
- reason mathematically, interpret and communicate mathematical information, make deductions and inferences, and draw conclusions;
- extend their base in mathematics from which they can progress to:
  - higher studies in mathematics; and/or
  - studies such as science, geography, technology or business, which contain a significant requirement in mathematics beyond Higher Tier GCSE Mathematics; and
- design and develop mathematical models that allow them to use problem-solving strategies and apply a broader range of mathematics to a variety of situations.

1.2 Key features

The following are important features of this specification.

- It offers opportunities to build on the skills and capabilities developed through the delivery of the Northern Ireland Curriculum at Key Stage 3.
- It caters for students who require knowledge of mathematics beyond GCSE Higher Tier Mathematics and who are capable of working beyond the limits of the GCSE Mathematics specification.
- It is designed to broaden the experience of students whose mathematical ability is above average and who would like to:
  - study mathematical courses at AS/A level;
  - study other courses at AS/A level that require mathematics beyond GCSE Higher Tier; or
  - extend their knowledge of mathematics.
- It gives students the appropriate mathematical skills, knowledge and understanding to help them progress to further academic and vocational study and to employment.
- This is a unitised specification. However, only Unit 1: Pure Mathematics is available in Summer 2018. Centres should be aware that first award is in Summer 2019. The legacy specification for GCSE Further Mathematics will be awarded for the last time in Summer 2018.

1.3 Prior attainment

Students taking this GCSE Further Mathematics specification should ideally have covered **all** of the content in the CCEA GCSE Mathematics specification at Higher Tier, including all of the content of units M4 and M8.

1.4 Classification codes and subject combinations

Every specification has a national classification code that indicates its subject area. The classification code for this qualification is 2330.

Please note that if a student takes two qualifications with the same classification code, schools, colleges and universities that they apply to may take the view that they have achieved only one of the two GCSEs. The same may occur with any two GCSE qualifications that have a significant overlap in content, even if the classification codes are different. Because of this, students who have any doubts about their subject combinations should check with the schools, colleges and universities that they would like to attend before beginning their studies.

2 Specification at a Glance

The table below summarises the structure of the GCSE Further Mathematics course. Students must complete the mandatory unit (Unit 1) and two of the three optional units (Units 2, 3 and 4).

Content	Assessment	Weightings	Availability
Unit 1: Pure Mathematics (Mandatory)	External written examination in the form of a single question-and-answer booklet that includes a formula sheet 2 hours	50%	Summer from 2018
Unit 2: Mechanics (Optional)	External written examination in the form of a single question-and-answer booklet that includes a formula sheet 1 hour	25%	Summer from 2019
Unit 3: Statistics (Optional)	External written examination in the form of a single question-and-answer booklet that includes a formula sheet 1 hour	25%	Summer from 2019
Unit 4: Discrete and Decision Mathematics (Optional)	External written examination in the form of a single question-and-answer booklet 1 hour	25%	Summer from 2019

Students must take at least 40 percent of the assessment (based on unit weightings) at the end of the course as terminal assessment.

3 Subject Content

We have divided this course into four units. The content of each unit and the respective learning outcomes appear below.

3.1 Unit 1: Pure Mathematics

In this unit, students investigate algebra, trigonometry, differentiation, integration, logarithms, matrices and quadratic inequalities.

Content	Learning Outcomes
	Students should be able to:
Algebraic fractions	• add, subtract, multiply and divide rational algebraic fractions with linear and quadratic numerators and/or denominators;
Algebraic manipulation	• manipulate algebraic expressions, including the expansion of three linear brackets;
Completing the square	• complete the square where the coefficient of x^2 will always be 1;
	• apply completing the square to solving quadratic equations and identifying minimum turning points;
Simultaneous equations	• form and solve three equations in three unknowns;
Quadratic inequalities	• solve quadratic inequalities, which are restricted to quadratic expressions that factorise;
Trigonometric equations	• sketch the graphs of sin x, cos x and tan x, where the range of x is a subset of $-360^o \leqslant x \leqslant 360^o$; and
	• solve simple trigonometric equations that lead to a maximum of two solutions in a given range.

Content	Learning Outcomes
	Students should be able to:
Differentiation	• differentiate expressions that are restricted to integer powers of x; • apply differentiation to: – gradients; – finding equations of tangents and normals at points on a curve; – simple optimisation problems; and – elementary curve sketching of a quadratic or cubic function;
Integration	• demonstrate knowledge that integration is the inverse process to differentiation; • integrate expressions that are restricted to integer powers of x, $(x \neq -1)$; • form and evaluate definite integrals; • apply integration to finding the area under a curve;
Logarithms	• demonstrate understanding of logarithms as a natural evolution from indices; • solve problems using: – the laws of logarithms; and – log/log graphs in context; • solve indicial equations using logarithms;
Matrices	• add, subtract and multiply matrices; • find the inverse of 2 × 2 matrices; • solve matrix equations; and • use matrices to solve 2 × 2 simultaneous equations.

3.2 Unit 2: Mechanics

In this unit, students explore kinematics, vectors, forces, Newton's Laws of Motion and moments.

Content	Learning Outcomes
	Students should be able to:
Kinematics	• draw, interpret and use displacement/time graphs and velocity/time graphs; • use constant acceleration formulae;
Vectors	• demonstrate understanding of the definition of vector and scalar quantities; • calculate the magnitude and direction of a vector; • use **i** and **j** vectors in calculations;
Forces	• demonstrate understanding that force is a vector; • identify all forces acting on a body; • resolve forces into components; • find the resultant of a set of forces;
Newton's laws of motion	• demonstrate understanding of and apply the concept of equilibrium; • apply F = ma to the following scenarios: – a body in horizontal or vertical motion; – a body on an inclined plane; and – two connected bodies in rectilinear motion; *(F = μR will not be tested – if included, friction will be given as a value or as a value per unit mass, where appropriate.)*
Moments	• demonstrate understanding of the Principle of Moments and equilibrium of a rigid body (restricted to a horizontal uniform rod supported by one or two pivots).

3.3 Unit 3: Statistics

In this unit, students investigate central tendency and dispersion, probability, the binomial and normal distributions and bivariate analysis.

Content	Learning Outcomes
	Students should be able to:
Central tendency and dispersion	• calculate the mean and standard deviation from data or estimates of these from grouped data; • calculate the mean and standard deviation for combined sets of data; • demonstrate knowledge of the effect on the mean and standard deviation of a linear transformation on a set of data;
Probability	• calculate combined probabilities using the addition rule, to include events that may not be mutually exclusive; • calculate and interpret conditional probabilities using expected frequencies, two-way tables, tree diagrams and Venn diagrams; • use the most appropriate method to solve complex problems, including the construction and use of Venn diagrams and tree diagrams;
Binomial distribution	• use Pascal's triangle to expand $(p + q)^n$ where $n \leqslant 8$; • understand and use the binomial expansion to calculate probabilities in real-life contexts;
Normal distribution	• recognise that the distribution of many real-world variables takes the shape of a bell curve; and • calculate a single probability from the normal distribution using tables and $z = \frac{(x - \mu)}{\sigma}$ where the mean and standard deviation are given.

Content	Learning Outcomes

Bivariate analysis	Students should be able to:
	• calculate and interpret Spearman's Rank Correlation Coefficient;
	  • draw the line of best fit by eye passing through $(\overline{x}, \overline{y})$; and
	  • calculate and use the equation of the line of best fit.

3.4 Unit 4: Discrete and Decision Mathematics

In this unit, students explore counting, logic, linear programming, time series and critical path analysis.

Content	Learning Outcomes
	Students should be able to:
Counting	• demonstrate understanding of and use the addition and multiplication principles to count events in series and parallel respectively; • calculate the number of ways of arranging r objects from n objects; • calculate the number of ways of choosing r objects from n objects;
Logic	• demonstrate understanding of the concept of Boolean variables, including forming compound expressions using logical operators AND, OR and NOT; • use truth tables to prove the equivalence of propositional statements (involving no more than three variables);
Linear programming	• model real-life scenarios as linear programming problems; • use graphical methods to maximise or minimise an expression involving up to five inequalities in one or two variables (solutions may be real numbers or restricted to integers);
Time series	• demonstrate understanding of why we smooth data; • calculate appropriate moving averages (using three, four or five points); and • draw a trend line and use it to make predictions (the plotting of original data will be given).

Content	Learning Outcomes

Critical path analysis	Students should be able to:
	• demonstrate understanding of how an activity network represents a project (using activities on an arc);
	  • construct an activity network from a precedence table;
	• identify the critical path by finding the earliest and latest event times;
	  • calculate float times; and
	  • perform basic scheduling using Gantt charts.

4 Scheme of Assessment

4.1 Assessment opportunities

For the availability of examinations, see Section 2.

This is a unitised specification; candidates must complete at least 40 percent of the overall assessment requirements at the end of the course, in the examination series in which they request a final subject grade. This is the terminal rule.

Candidates may resit individual assessment units once before cash-in. The better of the two results will count towards their final GCSE grade unless a unit is required to meet the 40 percent terminal rule. If it is, the more recent mark will count (whether or not it is the better result). Results for individual assessment units remain available to count towards a GCSE qualification until we withdraw the specification.

4.2 Assessment objectives

There are three assessment objectives for this specification (AO1, AO2 and AO3).

AO1 Use and apply standard techniques

Candidates should be able to:

- accurately recall facts, terminology and definitions;
- use and interpret notation correctly; and
- accurately carry out routine procedures or set tasks requiring multi-step solutions.

AO2 Reason, interpret and communicate mathematically

Candidates should be able to:

- make deductions, inferences and draw conclusions from mathematical information;
  - construct chains of reasoning to achieve a given result;
  - interpret and communicate information accurately;
  - present arguments and proofs; and
- assess the validity of an argument and critically evaluate a given way of presenting information.

AO3 Solve problems in mathematics and other contexts

Candidates should be able to:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
  - make and use connections between different parts of mathematics;
  - interpret results in the context of the given problem;
  - evaluate methods used and results obtained; and
- evaluate solutions to identify how they may have been affected by assumptions made.

4.3 Assessment objective weightings

The table below sets out the assessment objective weightings for each assessment component and the overall GCSE qualification.

Assessment Objective	Unit Weighting (%)				Overall Weighting (%)
	External Assessment				
	Unit 1	Unit 2	Unit 3	Unit 4	
AO1	35–45	35–45	35–45	35–45	40
AO2	25–35	25–35	25–35	25–35	30
AO3	25–35	25–35	25–35	25–35	30
Total Weighting	100	100	100	100	100

4.4 Reporting and grading

We report the results of individual assessment units on a uniform mark scale that reflects the assessment weighting of each unit. We determine the grades awarded by aggregating the uniform marks that candidates obtain in individual assessment units.

We award GCSE qualifications on a grade scale from A* to G, with A* being the highest. The nine grades available are as follows:

Grade	A*	A	B	C*	C	D	E	F	G

If candidates fail to attain a grade G or above, we report their result as unclassified (U).

4.5 Use of calculators

Candidates must use electronic calculators with trigonometric, logarithmic and relevant statistical functions. Please note that candidates must show the full development of their answers to obtain full credit.

Calculators must be:

- of a size suitable for use on the desk;
- either battery or solar powered; and
- free of lids, cases and covers which have printed instructions or formulae.

Calculators must **not**:

- be designed or adapted to offer any of these facilities:
  - language translators;
  - symbolic algebra manipulation;
  - symbolic differentiation or integration; or
  - communication with other machines or the internet;
- be borrowed from another candidate during an assessment by examination for any reason; or
  - have retrievable information stored in them, including (but not limited to):
  - databanks;
  - dictionaries;
  - mathematical formulae; or
  - text.

5 Grade Descriptions

Grade descriptions are provided to give a general indication of the standards of achievement likely to have been shown by candidates awarded particular grades. The descriptions must be interpreted in relation to the content in the specification; they are not designed to define that content. The grade awarded depends in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of candidates' performance in the assessment may be balanced by better performances in others.

Grade	Description
A	Candidates characteristically: • perform procedures accurately; • interpret and communicate complex information accurately; • make deductions and inferences and draw conclusions; • construct substantial chains of reasoning, including convincing arguments and formal proofs; • generate efficient strategies to solve complex mathematical and non-mathematical problems by translating them into a series of mathematical processes; • make and use connections, which may not be immediately obvious, between different parts of mathematics; and • interpret results in the context of the given problem.
C	Candidates characteristically: • perform routine single- and multi-step procedures effectively by recalling, applying and interpreting notation, terminology, facts, definitions and formulae; • interpret and communicate information effectively; • make deductions, inferences and draw conclusions; • construct chains of reasoning, including arguments; • generate strategies to solve mathematical and non-mathematical problems by translating them into mathematical processes, realising connections between different parts of mathematics; • interpret results in the context of the given problem; and • evaluate methods and results.

Grade	Description
F	Candidates characteristically: • recall and use notation, terminology, facts and definitions; • perform routine procedures, including some multi-step procedures; • interpret and communicate basic information; • make deductions and use reasoning to obtain results; • solve problems by translating simple mathematical and non-mathematical problems into mathematical processes; • provide basic evaluation of methods or results; and • interpret results in the context of the given problem.

6 Curriculum Objectives

This specification builds on the learning experiences from Key Stage 3 as required for the statutory Northern Ireland Curriculum. It also offers opportunities for students to contribute to the aim and objectives of the Curriculum at Key Stage 4, and to continue to develop the Cross-Curricular Skills and the Thinking Skills and Personal Capabilities. The extent of the development of these skills and capabilities will be dependent on the teaching and learning methodology used.

6.1 Cross-Curricular Skills at Key Stage 4

Communication
Students should be able to: • communicate meaning, feelings and viewpoints in a logical and coherent manner, *for example by using appropriate mathematical language and notation in response to open-ended tasks, problems, structured questions or examination questions*; • make oral and written summaries, reports and presentations, taking account of audience and purpose, *for example through the use of varied learning activities applied to a wide range of contexts that require students to organise and record data, justify choice of strategy to solve problems, articulate processes, proofs etc. and provide feedback from collaborative learning activities*; • participate in discussions, debates and interviews, *for example by sharing ideas, investigating misconceptions, exploring alternative strategies, justifying their choice of strategy, negotiating decisions and listening to others*; • interpret, analyse and present information in oral, written and ICT formats, *for example by developing a mathematical solution to a problem and communicating ideas, strategies and solutions*; and • explore and respond, both imaginatively and critically, to a variety of texts, *for example by using open-ended tasks and activities*.

Using Mathematics
Students should be able to: • use mathematical language and notation with confidence, *for example calculus, matrices and force diagrams*; • use mental computation to calculate, estimate and make predictions in a range of simulated and real-life contexts, *for example simplifying expressions in algebra*; • select and apply mathematical concepts and problem-solving strategies in a range of simulated and real-life contexts, *for example moving averages in a financial capability context or applying Newton's laws to questions relating to dynamics*; • interpret and analyse a wide range of mathematical data, *for example calculating an estimate for the mean and standard deviation from a grouped frequency distribution*; • assess probability and risk in a range of simulated and real-life contexts, *for example tree diagrams or Venn diagrams*; and • present mathematical data in a variety of formats that take account of audience and purpose, *for example numerical, graphical and algebraic representations*.
Using ICT
Students should be able to make effective use of information and communications technology in a wide range of contexts to access, manage, select and present information, including mathematical information, *for example calculators, suitable software packages to explore geometry, algebraic functions or calculus, researching data online, analysing data and working with formulae in spreadsheets*.

6.2 Thinking Skills and Personal Capabilities at Key Stage 4

Self-Management
Students should be able to: • plan work, *for example by identifying appropriate strategies, working systematically and persisting with open-ended tasks and problems*; • set personal learning goals and targets to meet deadlines, *for example by identifying, prioritising and managing actions required to develop competence and confidence in more challenging mathematics*; • monitor, review and evaluate their progress and improve their learning, *for example by self-evaluating their performance, identifying strengths and areas for improvement and seeking support where required*; and • effectively manage their time, *for example by planning, prioritising and minimising distractions in order to meet deadlines set by teachers*.

Working with Others

Students should be able to:

- learn with and from others through co-operation, *for example by engaging in discussions and explaining ideas, challenging and supporting one another, creating and solving each other's questions and working collaboratively to share methods and results during small group tasks*;
- participate in effective teams and accept responsibility for achieving collective goals, *for example by working together on challenging small group tasks with shared goals but individual accountability*; and
- listen actively to others and influence group thinking and decision-making, taking account of others' opinions, *for example by participating constructively in small group activities, articulating possible problem-solving strategies and presenting a well thought out rationale for one approach*.

Problem Solving

Students should be able to:

- identify and analyse relationships and patterns, *for example make links between cause and effects, for example by considering the rates of change of a function with respect to one of its variables through the study of calculus*;
- propose justified explanations, *for example by proving a mathematical statement is true*;
- analyse critically and assess evidence to understand how information or evidence can be used to serve different purposes or agendas, *for example using data to make predictions about the likelihood of future events occurring*;
- analyse and evaluate multiple perspectives, *for example modelling real-life scenarios as further mathematics problems*;
- weigh up options and justify decisions, *for example using the most appropriate method to solve complex mathematical problems*; and
- apply and evaluate a range of approaches to solve problems in familiar and novel contexts, *for example by choosing the appropriate diagrammatic method in probability or the use of constant acceleration formulae versus use of velocity/time graphs*.

There is an emphasis on problem solving in this specification, which will require the development of all of these skills.

Although not referred to separately as a statutory requirement at Key Stage 4 in the Northern Ireland Curriculum, **Managing Information** and **Being Creative** may also remain relevant to learning.

7 Links and Support

7.1 Support

The following resources are available to support this specification:

- our Mathematics microsite at www.ccea.org.uk and
- specimen assessment materials.

We also intend to provide:

- past papers;
- mark schemes;
- Chief Examiner's reports;
- planning frameworks;
- centre support visits;
- support days for teachers; and
- a resource list.

7.2 Examination entries

Entry codes for this subject and details on how to make entries are available on our Qualifications Administration Handbook microsite, which you can access at www.ccea.org.uk

Alternatively, you can telephone our Examination Entries, Results and Certification team using the contact details provided.

7.3 Equality and inclusion

We have considered the requirements of equality legislation in developing this specification and designed it to be as free as possible from ethnic, gender, religious, political and other forms of bias.

GCSE qualifications often require the assessment of a broad range of competences. This is because they are general qualifications that prepare students for a wide range of occupations and higher level courses.

During the development process, an external equality panel reviewed the specification to identify any potential barriers to equality and inclusion. Where appropriate, we have considered measures to support access and mitigate barriers.

We can make reasonable adjustments for students with disabilities to reduce barriers to accessing assessments. For this reason, very few students will have a complete barrier to any part of the assessment.

It is important to note that where access arrangements are permitted, they must not be used in any way that undermines the integrity of the assessment. You can find information on reasonable adjustments in the Joint Council for Qualifications document *Access Arrangements and Reasonable Adjustments*, available at www.jcq.org.uk

7.4 Contact details

If you have any queries about this specification, please contact the relevant CCEA staff member or department:

- Specification Support Officer: Nuala Tierney
  (telephone: (028) 9026 1200, extension 2292, email: ntierney@ccea.org.uk)

- Subject Officer: Gavin Graham
  (telephone: (028) 9026 1200, extension 2658, email: ggraham@ccea.org.uk)

- Examination Entries, Results and Certification
  (telephone: (028) 9026 1262, email: entriesandresults@ccea.org.uk)

- Examiner Recruitment
  (telephone: (028) 9026 1243, email: appointments@ccea.org.uk)

- Distribution
  (telephone: (028) 9026 1242, email: cceadistribution@ccea.org.uk)

- Support Events Administration
  (telephone: (028) 9026 1401, email: events@ccea.org.uk)

- Moderation
  (telephone: (028) 9026 1200, extension 2236, email: moderationteam@ccea.org.uk)

- Business Assurance (Complaints and Appeals)
  (telephone: (028) 9026 1244, email: complaints@ccea.org.uk or appealsmanager@ccea.org.uk).

Summary of Changes since First Issue

(Most recent changes are indicated in red on the latest version)

Revision History Number	Date of Change	Page Number	Change Made
Version 1	N/A	N/A	First issue
Version 2	26 June 2017	23	Contact details updated
Version 2	17 September 2019	23	Contact details updated