Rewarding Learning

General Certificate of Secondary Education
January 2019

Further Mathematics

Unit 2
Mechanics and Statistics

[GMF21]

WEDNESDAY 23 JANUARY, AFTERNOON

MARK SCHEME

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

MW indicates marks for combined method and accurate working.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

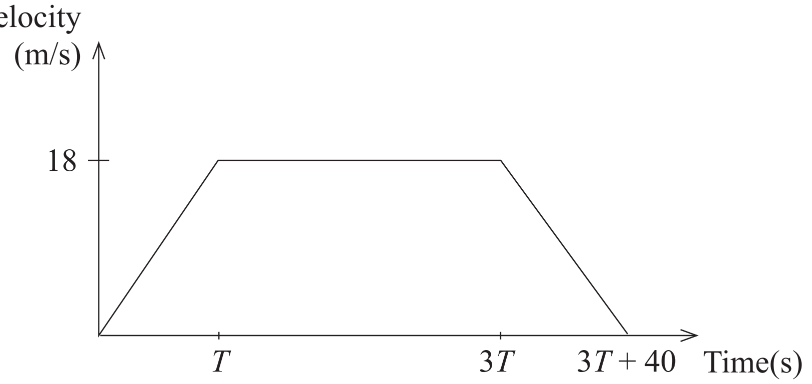
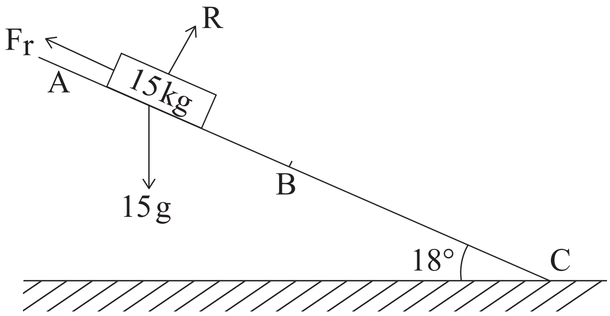
(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

				AVAILABLE MARKS
1	**(i)**	$u = -\mathbf{i} - 13\mathbf{j}$, $t = 4$, $v = -5\mathbf{i} + 23\mathbf{j}$		
		$v = u + at$		
		$-5\mathbf{i} + 23\mathbf{j} = -\mathbf{i} - 13\mathbf{j} + 4a$	MW1	
		$a = -\mathbf{i} + 9\mathbf{j}$ m/s^2	W1	
	(ii)	$\mathbf{F} = \text{m}a$		
		$\mathbf{F} = 2.5 \times (-\mathbf{i} + 9\mathbf{j})$		
		$\mathbf{F} = (-2.5\mathbf{i} + 22.5\mathbf{j})$ N	MW1	
		Magnitude of $\mathbf{F} = \sqrt{(-2.5)^2 + 22.5^2}$	M1	
		Magnitude of $\mathbf{F} = 22.64$ N	W1	5
2	**(i)**	$a = -10$, $s = 12.8$, $v = 0$		
		$v^2 = u^2 + 2as$		
		$0 = u^2 + 2 \times (-10) \times 12.8$	MW1	
		$u^2 = 256$		
		$u = \sqrt{256} = 16$ m/s	W1	
	(ii)	$a = -10$, $s = -1.4$, $u = 16$		
		$s = ut + \frac{1}{2}at^2$		
		$-1.4 = 16t + \frac{1}{2}(-10)t^2$	M1	
		$5t^2 - 16t - 1.4 = 0$	MW1	
		$t = \dfrac{16 \pm \sqrt{16^2 - 4(5)(-1.4)}}{10}$	MW1	
		$t = 3.29$ s	W1	

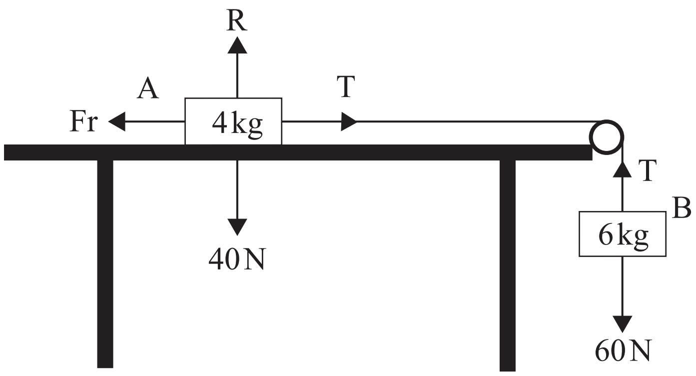
Alternative solution

Way up		**Way down**		
$a = -10$, $u = 16$, $v = 0$		$a = 10$, $u = 0$, $s = 14.2$		
$v = u + at$		$s = ut + \frac{1}{2}at^2$		
$0 = 16 + (-10)t$	M1	$14.2 = \frac{1}{2} \times (10) \times t^2$		
$t = 1.6$	MW1	$t^2 = 2.84$		
		$t = 1.685$	MW1	
	Total time = 1.6 + 1.685 = 3.29 s		W1	6

				AVAILABLE MARKS
3	**(i)**		W1 (trapezium shape) W1 (correct values on axes)	
	(ii)	Area under graph $\frac{1}{2}(3T + 40 + 2T) \times 18 = 900$ $45T + 360 = 900$ $T = 12$	 M1 W1 W1	
	(iii)	Acceleration $= \frac{18 - 0}{12}$ $= 1.5\,\text{m/s}^2$	 MW1	6
4	**(i)**	Resolving vertically $25 + R_D = 80$ $R_D = 55\,\text{N}$	 MW1 W1	
	(ii)	Take moments about A and take d as the distance AD $25 \times 0.6 + R_D \times d = 80 \times 2$ $15 + 55d = 160$ $d = \frac{29}{11} = 2.64\,\text{m}$	 MW1 MW1 W1	
	(iii)	Take moments about C $300 \times 0.6 = 80 \times 1.4 + 10M \times 3.4$ $M = 2$	 MW1 MW1 W1	8
5	**(i)**		W2 (all 3 correct) [W1 (any 2 correct)]	
	(ii)	$u = 9, v = 7, s = 4$ $v^2 = u^2 + 2as$ $49 = 81 + 8a$ $a = -4\,\text{m/s}^2$ deceleration $= 4\,\text{m/s}^2$	 MW1 W1	

		AVAILABLE MARKS
(iii) Resolve perpendicular to ramp:		
$R = 15g \cos 18°$	MW1	
$R = 142.658\text{ N}$		
Resolve parallel to ramp:		
$15g \sin 18° - F_r = 15(-4)$	MW1 MW1	
$F_r = 15g \sin 18° + 60$		
$F_r = 106.353\text{ N}$	W1	
$F_r = \mu R$		
$\mu = \frac{106.353}{142.658} = 0.75$	MW1	
(iv) Motion from A to C:		
$u = 9,\ v = 0,\ a = -4$		
$v^2 = u^2 + 2as$		
$0 = 81 - 8s$	MW1	
$s = \frac{81}{8} = 10\frac{1}{8} = 10.125\text{ m}$	W1	
Alternative solution		
Motion from B to C:		
$u = 7,\ v = 0,\ a = -4$		
$v^2 = u^2 + 2as$		
$0 = 49 - 8s$	MW1	
$s = 6\frac{1}{8}$		
Length of ramp $= 4 + 6\frac{1}{8} = 10\frac{1}{8} = 10.125\text{ m}$	W1	11

6 (i)



All 6 forces correct W2
(Any 3 correct W1)

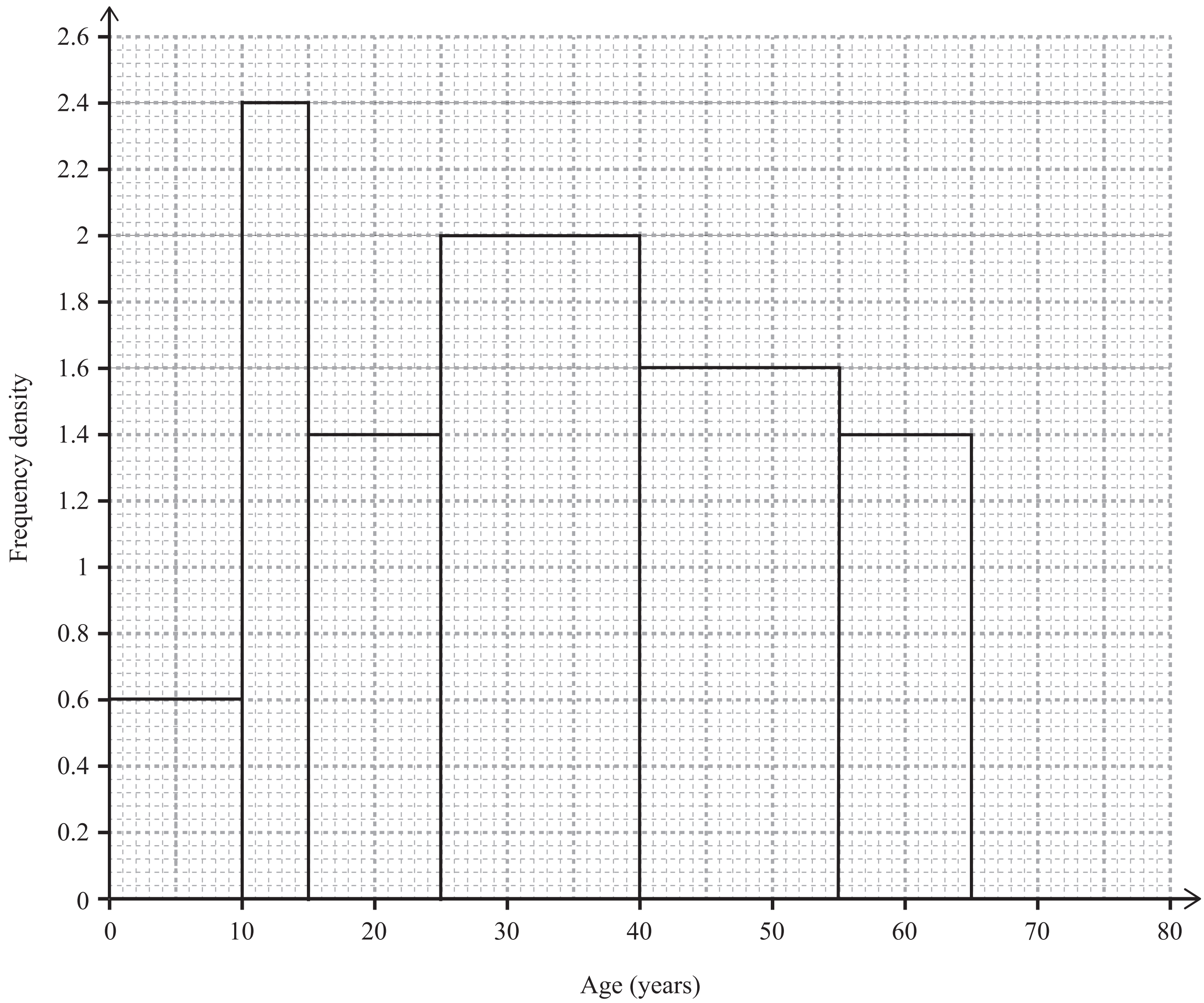
(ii) Block A vertically: R = 40, so max friction = 0.4 × 40 = 16 N	MW1	
Block A horizontally: $T - 16 = 4a$	MW1	
Block B: $60 - T = 6a$	MW1	
Adding: $44 = 10a$		
$a = \frac{44}{10} = 4.4\text{ m/s}^2$	W1	
(iii) $60 - T = 6 \times 4.4$	MW1	
$T = 33.6\text{ N}$	W1	
(iv) $u = 0,\ t = 1.4,\ a = 4.4$		
$v = u + at$		
$v = 0 + 1.4 \times 4.4$	MW1	
$v = 6.16\text{ m/s}$	W1	

			AVAILABLE MARKS
	(v) Using $F = ma$ $0 - 16 = 4a$ $a = -4\,\text{m/s}^2$ $u = 6.16$, $v = 0$, $a = -4$ Using $v^2 = u^2 + 2as$ $0 = 6.16^2 - 8s$ $s = 4.7432\,\text{m} \rightarrow 4.74\,\text{m}$	 MW1 M1 MW1 W1	 14
7	**(i)** 12.5 – 14.4	MW1	
	(ii) 14.45 – 16.45	MW1	
	(iii) 2	MW1	3
8	Median $= 24.5 + \dfrac{(30 - 9) \times 10}{26}$ $= 32.58$ (or 32.77 if 30.5 used instead of 30)	M1 (24.5 + ...) MW1 (30 – 9) MW1 (10/26) W1	 4

AVAILABLE MARKS

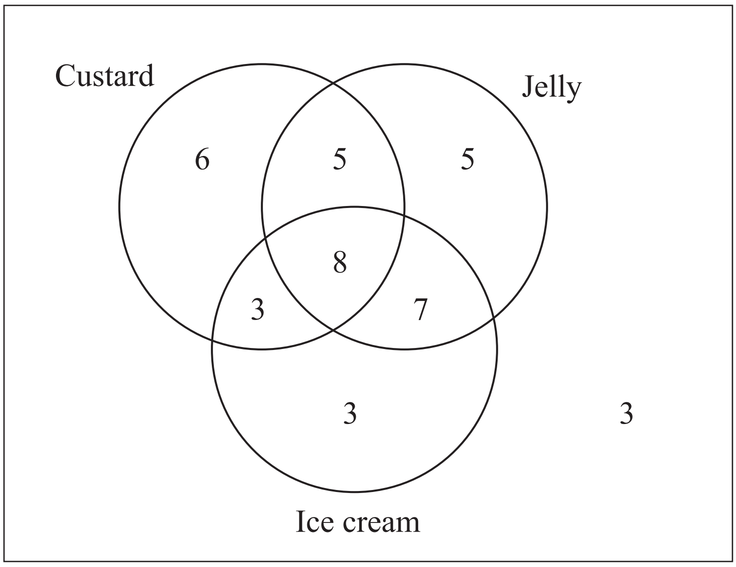
9 Frequency densities $\frac{6}{10}, \frac{12}{5}, \frac{14}{10}, \frac{30}{15}, \frac{24}{15}, \frac{14}{10}$

$= 0.6, 2.4, 1.4, 2.0, 1.6, 1.4$ M1 W1



W1 (labels)
MW1 (heights)
MW1 (boundaries) 5

10 (i)



3 × MW1

(ii) 40 – (6 + 5 + 5 + 3 + 8 + 7 + 3) = 3 MW1

(iii) 5 + 3 + 7 = 15

P (exactly 2) = $\frac{15}{40}$ or $\frac{3}{8}$ MW1 W1

(iv) 18 did not choose custard MW1
Chose jelly, but not custard = (5 + 7) = 12 MW1
P (jelly | not custard) = $\frac{12}{18}$ or $\frac{2}{3}$ MW1

AVAILABLE MARKS: 9

11 (i)

Ranks (Height)	7	9	1	3	2	4	6	5	8
Ranks (Mass)	6	9	1	4	2	4	7	4	8

or

Ranks (Height)	3	1	9	7	8	6	4	5	2
Ranks (Mass)	4	1	9	6	8	6	3	6	2

MW1 MW1

(ii)

d^2	1	0	0	1	0	0	1	1	0

M1 W1

$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$

$r = 1 - \frac{6(4)}{9(80)}$ M1

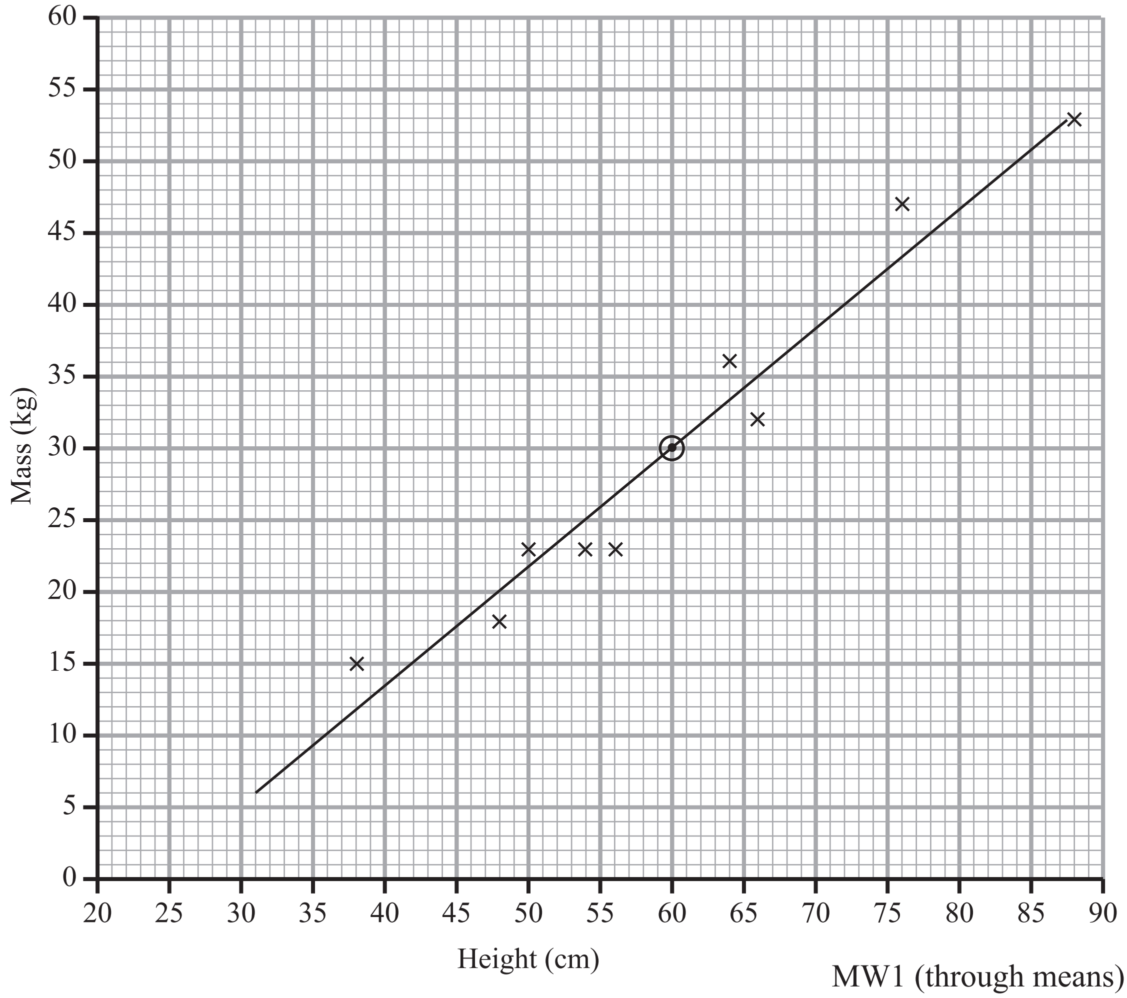
r = 0.97 W1

(iii) Strong positive correlation M1

(iv) Mean height = $\frac{540}{9}$ = 60 cm

Mean mass = $\frac{270}{9}$ = 30 kg MW1

(v)



MW1 (through means)
W1 (slope)

(vi) Gradient = $\frac{51-30}{85-60} = 0.84$ MW1

Using means
$30 = 0.84(60) + c$ M1
$c = -20.4$
Equation is
$y = 0.84x - 20.4$ MW1

AVAILABLE MARKS: 13

		AVAILABLE MARKS
12 **(i)** Mean = $\frac{6.5 \times 8 + 5.5 \times 12}{20} = 5.9$	MW1 W1	
(ii) For the girls		
$2.14^2 = \frac{\sum g^2}{8} - 6.5^2$	MW1	
$\sum g^2 = 374.6368$		
For the boys		
$1.52^2 = \frac{\sum b^2}{12} - 5.5^2$	MW1	
$\sum b^2 = 390.7248$		
For all		
$s^2 = \frac{374.6368 + 390.7248}{20} - 5.9^2$	MW1	
$s = 1.86$	W1	6
13 **(i)** P(<200) = 0.5	MW1	
(ii) P(>250) = 0.25	MW1	
(iii) $\text{P}(>250 \mid >100) = \frac{\text{P}(>250)}{\text{P}(>100)}$	M2	
$= \frac{0.25}{0.75} = \frac{1}{3}$	W1	5
14 Jack: Prob WW = $\frac{5}{7} \times \frac{5}{7} = \frac{25}{49} = 0.51$	M1 W1	
Jill: Prob WW = $\frac{10}{14} \times \frac{9}{13} = \frac{90}{182} = 0.49$	M1 W1	
Jack has greater chance of picking 2 white balls	M1	5
	Total	**100**