Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2019

Mathematics

Assessment Unit F1
assessing
Module FP1: Further Pure Mathematics 1

AMF11

[AMF11]
MONDAY 20 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Let $\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(i) Verify that $\mathbf{A}^2 = 4\mathbf{A} - 5\mathbf{I}$ [4]

(ii) Hence, or otherwise, express the matrix $\mathbf{A}^4$ in the form $\alpha\mathbf{A} + \beta\mathbf{I}$, where α, β are integers. [4]

2 The equations of two circles C_1 and C_2 are

$$C_1 \qquad x^2 + y^2 - 10y + 16 = 0$$

$$C_2 \qquad x^2 + y^2 - 6x - 16 = 0$$

Find the exact coordinates of the points of intersection of C_1 and C_2 [8]

3 **(i)** Find the 2×2 matrix $\mathbf{M}$ which represents a reflection in the line $y = -\sqrt{3}\,x$ [3]

The transformation represented by the 2×2 matrix $\mathbf{N} = \begin{pmatrix} 3 & a \\ b & 4 \end{pmatrix}$ maps the points

$(2, a)$ and $(5, b + 2)$ onto $(7, 0)$ and $(15, -10)$ respectively.

(ii) Find the values of a and b. [6]

The matrix $\mathbf{S}$ represents the combined effect of the transformation represented by $\mathbf{M}$ followed by the transformation represented by $\mathbf{N}$

(iii) Find the matrix $\mathbf{S}$ [3]

4 The matrix $\mathbf{P} = \begin{pmatrix} 3 & -4 & 2 \\ -4 & -1 & 6 \\ 2 & 6 & -2 \end{pmatrix}$

(i) Show that 3 is an eigenvalue of **P** and find the other two eigenvalues. [6]

(ii) Find an eigenvector corresponding to the eigenvalue 3 [5]

(iii) Verify that $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$ are eigenvectors of **P** [3]

(iv) If **U** is a 3 × 3 matrix such that

$$\mathbf{U}^{-1}\,\mathbf{P}\,\mathbf{U} = \mathbf{D}$$

write down a possible diagonal matrix **D** [1]

5 **(a)** The group G consists of the elements $\{e, a, r, ar\}$ under multiplication.
e is the identity
$a^2 = r^2 = e$
$ar = ra$

(i) Show that $rar = a$ [2]

(ii) Draw up the multiplication table for group G [2]

The multiplication table for group H is shown below.

$\times$	e	p	q	s
e	e	p	q	s
p	p	q	s	e
q	q	s	e	p
s	s	e	p	q

(iii) Explain why the groups G and H are not isomorphic. [1]

(b) A binary operation $*$ is defined on the set of all ordered pairs (x, y) by

$$(a, b) * (c, d) = (ac + a + c, bd)$$

where x, y are real numbers such that $x \neq -1$ and $y \neq 0$

(i) Show that $*$ is commutative. [2]

(ii) Find the identity element, clearly showing all working. [4]

(iii) Find the inverse of (a, b). [4]

6 **(a)** On a clearly labelled Argand diagram, sketch the locus of those points w which satisfy

$$1 < |w - (2 + 4i)| < 4$$ [4]

(b) **(i)** Given that

$$(p + qi)^2 = -80 + 18i$$

where p and q are real numbers,
find the values of p and corresponding values of q. [8]

(ii) Hence solve the quadratic equation

$$z^2 + (-1 + 5i)z + 7(2 - i) = 0$$ [5]

THIS IS THE END OF THE QUESTION PAPER
