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ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2019

Further Mathematics

Assessment Unit AS 2
assessing
Applied Mathematics

SFM21

[SFM21]

MONDAY 17 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
You must answer **all** questions from sections A and B **or** A and C **or** A and D **or** C and D.
You should spend equal time on each of the two sections.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
The total mark for each section of this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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SECTION A Mechanics 1

Answer all four questions in this section.

1 **(a)** A force

$$\mathbf{F} = (7\mathbf{i} - 2\mathbf{j} + \mathbf{k})\,\text{N}$$

moves a particle from a point A to a point B where

$$\overrightarrow{\text{OA}} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k})\,\text{m}$$

$$\overrightarrow{\text{OB}} = (5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})\,\text{m}$$

Find the work done by the force. [4]

(b) A particle P is moving along a horizontal line which passes through a fixed point O.
The distance from O to P is x metres, where $x > 0$
A variable force F newtons acts on P where

$$F = 8x - \frac{18}{x^3}$$

(i) Find the work done by F, in terms of d, when the particle P moves from $x = 1.5$ to $x = d$, where $d > 1.5$ [4]

The particle P has mass 3 kg.
It is at rest when $x = 1.5$ and has velocity $4\,\text{m}\,\text{s}^{-1}$ when $x = d$

(ii) Find the value of d. [4]

2 A child's toy takes the form of a smooth hemispherical bowl fixed to a horizontal surface at point B as shown in **Fig. 1** below.

A small plastic mouse M moves in a horizontal circle inside the bowl.

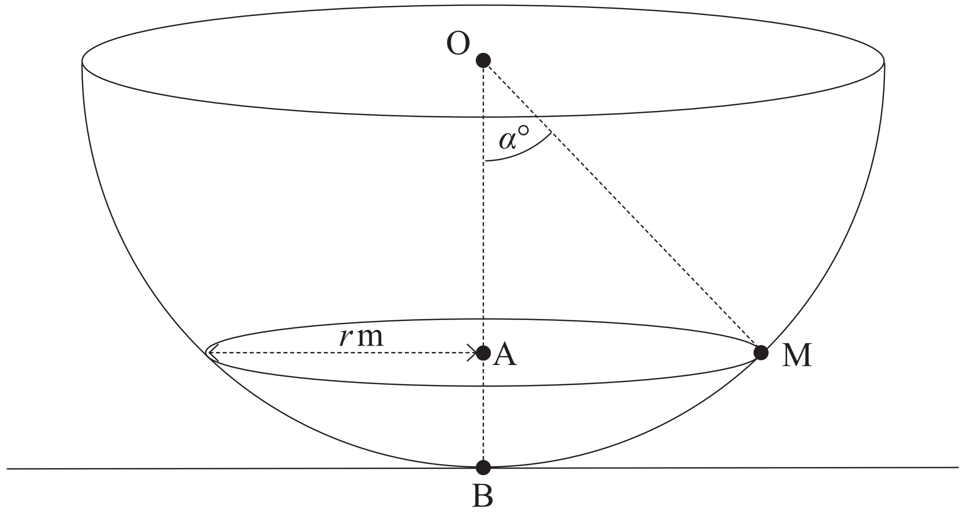

Fig. 1

The centre of the bowl is O.
The angle between OM and the vertical OB is $\alpha°$
The mouse moves in a horizontal circle with centre A and radius r m.

Model the mouse as a particle of mass m kg, travelling at a constant speed $v\,\text{m s}^{-1}$

(i) Explain why the normal reaction must act through O. [1]

(ii) Draw a diagram to show all the forces acting on M. [2]

(iii) Show that

$$v^2 = gr\tan\alpha$$ [7]

The radius of the bowl is 40 cm and AB is 8 cm.

(iv) Find the speed with which the mouse moves. [2]

3 A pump lifts still water through a pipe to a vertical height of 25 metres where it is moving at $5\,\text{m s}^{-1}$
The pipe has a circular cross section with diameter 0.28 m.
The density of water is $1000\,\text{kg m}^{-3}$

Find the effective power of the pump. [9]

4 A large catapult is formed by joining two identical light elastic strings to fixed points A and B which are 1 m apart on the same horizontal level as shown in **Fig. 2** below.

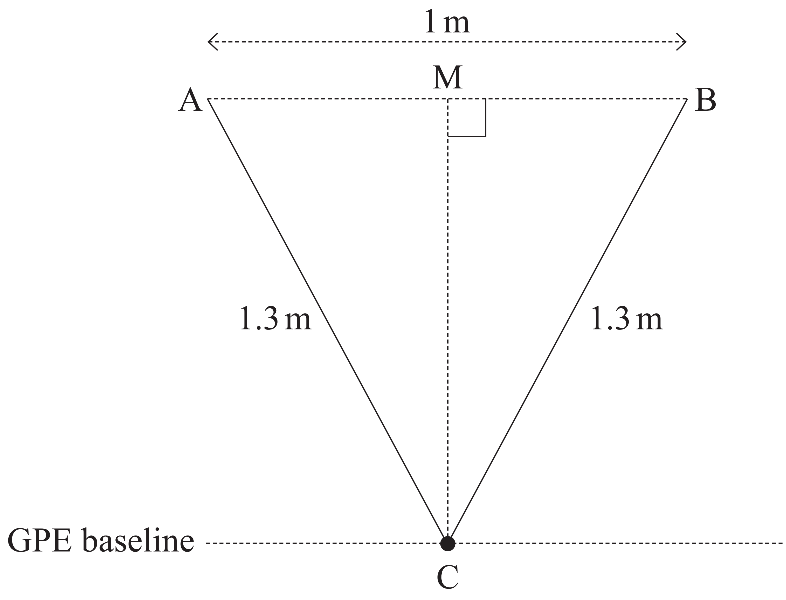

Fig. 2

Each string has a natural length of 0.6 m and modulus of elasticity 400 N.
Both free ends are attached to a pan of mass 1 kg.
A ball of mass 9 kg is placed freely in the pan.

The pan is pulled vertically downwards to the point C at ground level where AC = BC = 1.3 m.
Consider the pan and ball as particles.
Take ground level to be the baseline for gravitational potential energy.

(i) Find the energy of the system when the pan and ball are being held at C before their release. [4]

The pan is then released from rest.
The ensuing motion of the pan and the ball takes place along the line MC, the perpendicular bisector of AB.

(ii) Using the Principle of Conservation of Mechanical Energy, find the velocity of the pan and ball at the instant when the strings go slack. [9]

(iii) Find the greatest height of the ball above ground level in the subsequent motion. [4]

SECTION B Mechanics 2

Answer all five questions in this section.

1 **Fig. 1** below shows two light strings AP and BP.
These are used to keep a particle of mass 2 kg in equilibrium at P.
The ends A and B are attached to fixed points.

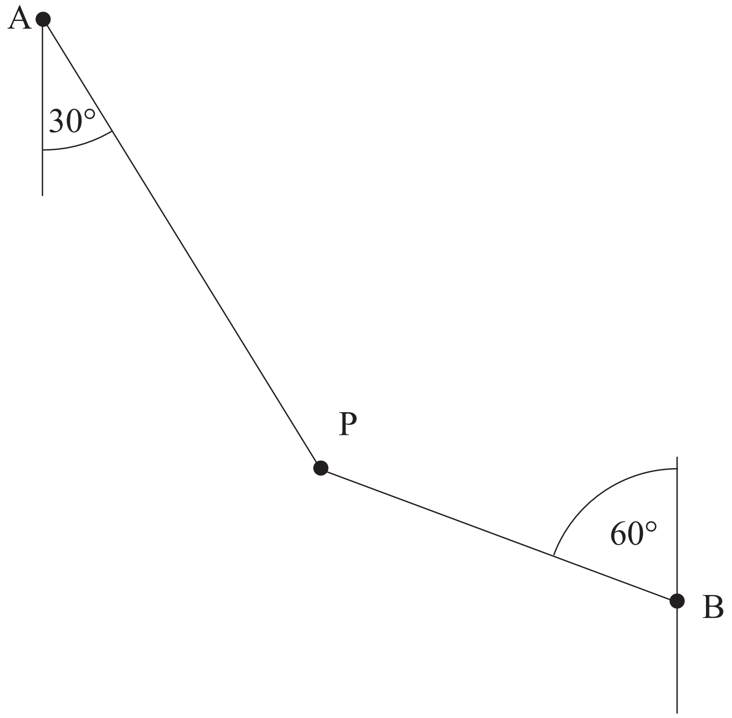

Fig. 1

The string AP is elastic with modulus of elasticity λ newtons and natural length 40 cm.
It is inclined at 30° to the vertical and has an extension of 10 cm.

The string BP is inextensible and is inclined at 60° to the vertical.

Find the value of λ. [8]

2 The viscosity of a fluid η is related to the force F required to move it at speed s by the formula

$$F = \frac{\eta A s}{y}$$

where A is an area and y is a distance.

(i) Use the Method of Dimensions to find the dimensions of viscosity. [4]

(ii) Poiseuille's law of viscous fluid motion assumes that the change of fluid volume per unit time, Q, along a pipe depends on:

R the radius of the pipe,

L the length of the pipe,

P the pressure difference, where pressure is the force per unit area, and

η the viscosity of the fluid.

Assuming a product relationship of the form

$$Q = kR^a L^b P^c \eta^d$$

where k is a dimensionless constant, use the Method of Dimensions to find the values of c and d and the relationship between a and b. [7]

3 Kepler's third law of planetary motion can be stated in the form

$$T^2 = kR^3$$

where T is the period of a planet's orbit about its sun,
R is the distance between the centres of the planet and the sun and
k is a constant of proportionality.

Assume the orbit is circular.

Take: M_p as the mass of the planet,
M_s as the mass of the sun, and
G as the Universal Gravitational Constant.

By considering the motion of the planet around the sun, derive Kepler's third law, stating k in terms of M_s and G. [8]

4 Take **i** to be a unit vector in the direction East and **j** to be a unit vector in the direction North.
A private yacht A is travelling with constant velocity $(18\mathbf{i} - 5\mathbf{j})\,\text{km h}^{-1}$
A coastguard boat B is travelling with constant velocity $(12\mathbf{i} + 16\mathbf{j})\,\text{km h}^{-1}$
At 1600 hours B is 12 km due South of A.

(i) Find the shortest distance between A and B in the subsequent motion. [7]

(ii) Find the time, to the nearest minute, when B is at its shortest distance from A. [4]

5 Designers of a snow park plan a ski jump and landing as shown in **Fig. 2** below.

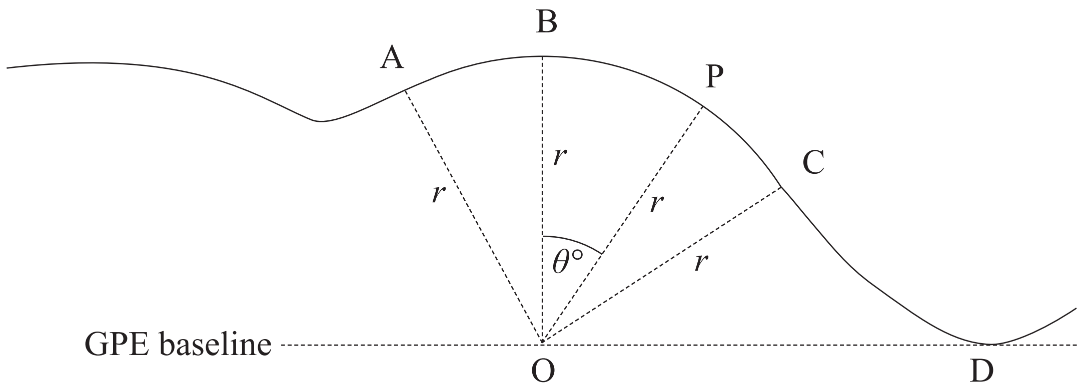

Fig. 2

The section of the jump between points A and C is an arc of a vertical circle, radius r, centred at the point O on the same horizontal level as D.
B is the highest point of this circle.

Model the skier as a particle of mass m kg and assume the snow is a smooth surface.
A skier on the jump reaches point D with speed $u\ \text{m s}^{-1}$
Take the gravitational potential energy baseline to be the horizontal line through O and D.

(i) Find an expression for the velocity, $v\ \text{m s}^{-1}$, of the skier at a point P on the arc BC, in terms of u, r and θ, where angle POB is $\theta°$ [5]

(ii) Show that the normal reaction N, between the skier and the slope at the point P, is given by

$$\text{N} = 3mg\cos\theta - \frac{mu^2}{r}$$ [5]

(iii) If $u = 21\ \text{m s}^{-1}$ and $r = 10\sqrt{3}$ m, find the value of θ when the skier starts to leave the surface of the snow. [2]

SECTION C Statistics

Answer all five questions in this section.

1 The speed of a car, v miles per hour, at time t seconds after it starts to accelerate is shown in **Table 1** below.

Table 1

Time, t	20	30	40	50	60	70	80	90
Speed, v	40	51	63	73	82	89	95	102

(i) Explain briefly what is meant by the terms explanatory and response in relation to these variables of speed and time. [2]

(ii) Calculate the equation of the regression line of speed v on time t. [6]

(iii) Estimate the value of v when $t = 65$ [2]

(iv) Explain why it would not be appropriate to estimate the speed for the car at a time of 120 seconds. [1]

2 The continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = kx(4 - x) \quad \text{for } 0 \leqslant x \leqslant 2$$

(i) Show that the value of k is $\frac{3}{16}$ [4]

(ii) Find the variance of X. [5]

(iii) Find the modal value of X. [4]

3 Accidents occur on a particular road junction at an average rate of 1.75 per week during the months of December, January and February.

(i) Find the probability that exactly three accidents occur during a particular week in January. [2]

(ii) Find the probability that exactly three accidents will occur in each of two successive weeks in January. [2]

The council have decided that if there are at least four accidents during the first two weeks of December, they will introduce new traffic lights.

(iii) Find the probability that they will have to introduce the new traffic lights after these particular two weeks. [5]

4 A Human Resources Department within a large manufacturing company wishes to assess how satisfied the employees are with their working conditions.
It is decided to question a sample of employees in order to obtain their opinions.

(i) Describe a suitable sampling technique that they could use. [3]

(ii) State an advantage and a disadvantage of the sampling technique you have chosen. [2]

5 There are ten members in the local swimming club, four males and six females.
Four members are to be chosen at random to represent the club at a charity swim event.

(i) How many different selections are possible? [2]

(ii) How many of these selections will include both the fastest male swimmer and the fastest female swimmer? [2]

(iii) Write down the probability that the fastest male and the fastest female swimmers will be chosen. [2]

Before the participants are selected, it is decided that two male swimmers and two female swimmers are to be chosen for the event.

Two male members and two female members are selected at random.

(iv) How many different selections are possible? [3]

(v) Find the probability that both the fastest male swimmer and the fastest female swimmer are now chosen. [3]

SECTION D Discrete and Decision Mathematics

Answer all five questions in this section.

1 **(a)** **(i)** Explain what is meant by a Hamiltonian cycle in a graph. [2]

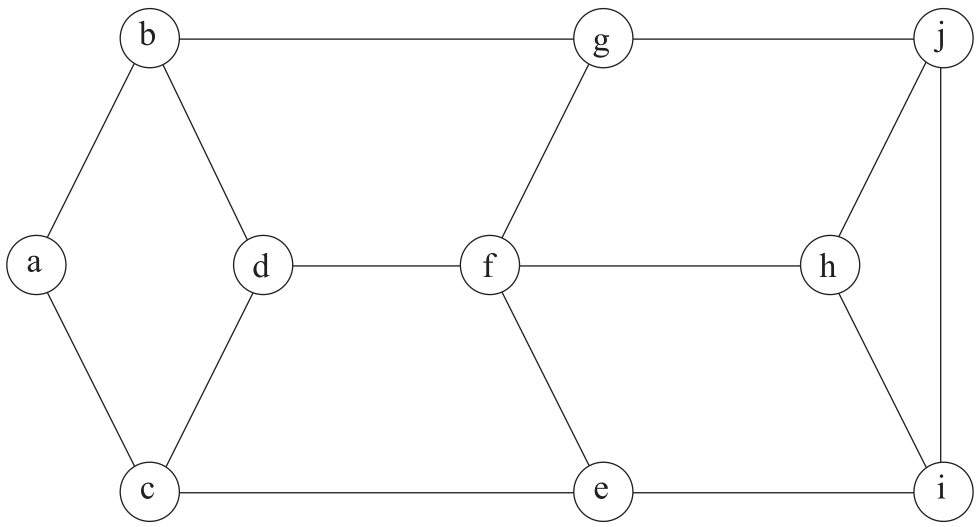

Fig. 1

(ii) Write down a Hamiltonian cycle for the graph in **Fig. 1** above. [2]

(b) **(i)** Explain what is meant by an Eulerian circuit in a graph. [2]

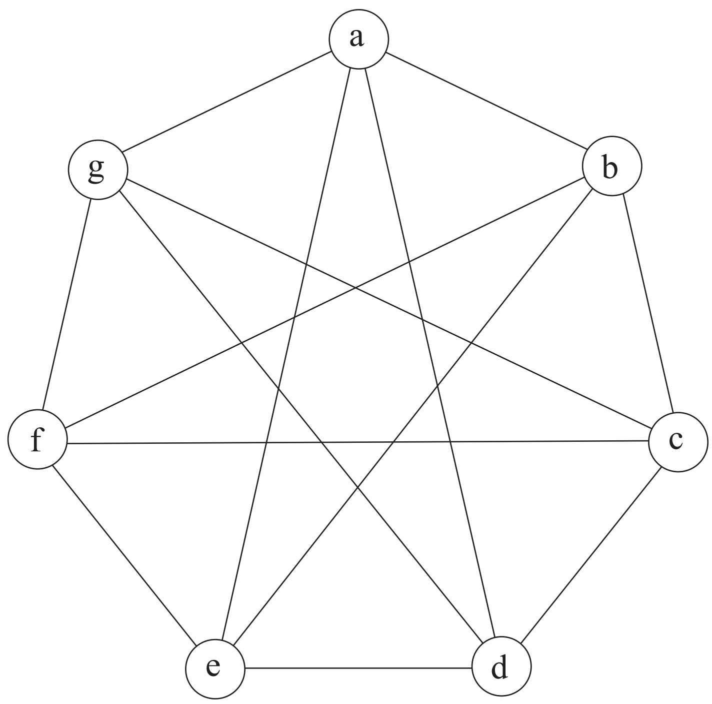

Fig. 2

(ii) Write down an Eulerian circuit for the graph in **Fig. 2** above. [2]

2 The Pell sequence is defined by the recurrence relationship

$$P_{n+2} = 2P_{n+1} + P_n \qquad\qquad n \geqslant 0$$

with the starting values $P_0 = 0$ and $P_1 = 1$

(i) Show that the auxiliary equation is

$$x^2 - 2x - 1 = 0$$ [2]

(ii) Hence, show that

$$P_n = \frac{1}{2\sqrt{2}}\left(\alpha^n - \beta^n\right)$$

where $\alpha = 1 + \sqrt{2}$ and $\beta = 1 - \sqrt{2}$ [7]

3 Let p, q and r be propositional statements.

Use truth tables to prove:

p and (r or $\sim q$) $\equiv$ (p and r) or (p and $\sim q$) [8]

4 **(a)** **Fig. 3** below shows the complete graph on 6 vertices, K_6

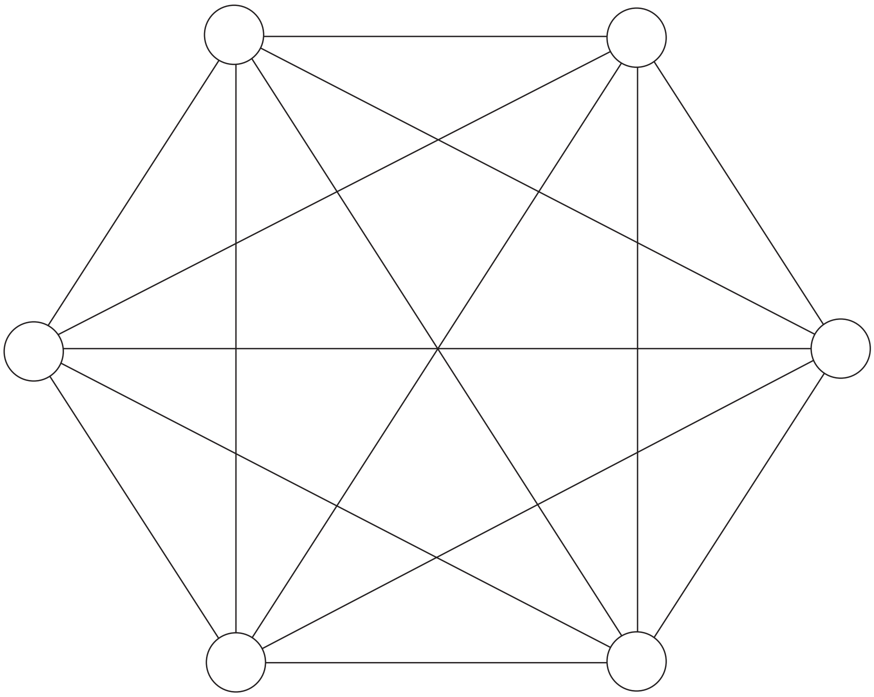

Fig. 3

How many edges must be removed to leave a spanning tree? [3]

(b) **Fig. 4** below shows distances in kilometres between six towns A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	129	175	86	84	232
B	129	–	211	119	212	235
C	175	211	–	148	160	147
D	86	119	148	–	85	190
E	84	212	160	85	–	200
F	232	235	147	190	200	–

Fig. 4

(i) Using Prim's algorithm, starting from A, find a minimum spanning tree for this graph.
Carefully list the edges in the order that they are selected. [5]

(ii) What is the total distance of this minimum spanning tree? [1]

5 An incomplete Cayley table for the order 8 group (Q, ⊗) is given in **Fig. 5** below.

⊗	A	B	C	D	E	F	G	H
A	A	B	C	D	E	F	G	H
B	B	A	D	C		E	H	
C	C	D	B		G	H	F	
D	D	C	A	B				F
E		F	H	G		A	C	D
F	F	E		H	A	B		C
G		H	E	F	D		B	A
H	H		F		C	D	A	B

Fig. 5

(i) Using C⊗E = G and operating on the left by B, prove that D⊗E = H [2]

(ii) Copy and complete the Cayley table using the Latin Square property. [3]

(iii) Find the period of each element of (Q, ⊗) [4]

The cyclic group (R, ✶) of order 8 has generator g and identity e.

The elements of (R, ✶) and their periods are given in **Fig. 6** below.

Element	e	g	g^2	g^3	g^4	g^5	g^6	g^7
Period	1	8	4	8	2	8	4	8

Fig. 6

(iv) Are the groups (R, ✶) and (Q, ⊗) isomorphic? Justify your answer. [2]

(v) List a subgroup of order 4 of (R, ✶) and a subgroup of order 4 of (Q, ⊗) which are isomorphic, showing them to be isomorphic by writing down a valid mapping from Q to R. [5]

THIS IS THE END OF THE QUESTION PAPER

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