New Specification

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ADVANCED SUBSIDIARY (AS)

General Certificate of Education

2019

Further Mathematics

Assessment Unit AS 1

assessing

Pure Mathematics

SFM11

[SFM11]

WEDNESDAY 12 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(a)** Given $\mathbf{A} = \begin{pmatrix} 3 & 7 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$

find:

(i) $\mathbf{AB}$ [2]

(ii) $(\mathbf{AB})^{-1}$ [4]

(iii) Show that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\,\mathbf{A}^{-1}$ [4]

(b) A transformation is defined by $\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

This transformation maps any point on the line $y = mx$ onto another point on the line $y = mx$.

Find the possible values of m. [6]

2 **Fig. 1** below shows a parallelepiped ABCDEFGH.

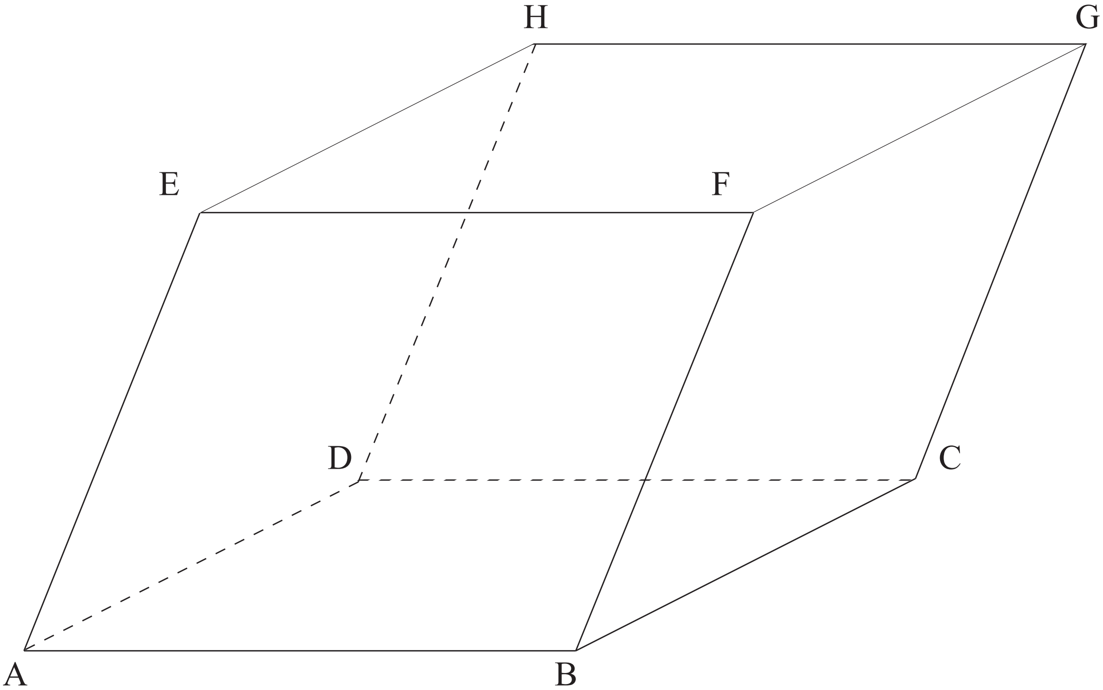

Fig. 1

The vertices A, B, D, E have coordinates (0, 0, 0), (5, −2, 3), (2, −3, 4) and (3, −2, −3) respectively.

Find the volume of the parallelepiped. [6]

3 **(a)** **(i)** Find the inverse of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 7 & -4 \\ 1 & 4 & -2 \\ -1 & -5 & 3 \end{pmatrix}$$ [4]

(ii) Find the matrix **B** which satisfies the equation

$$\mathbf{A}^2 + \mathbf{AB} = \mathbf{I}$$ [3]

(b) The roots of the equation $2x^2 + 5x - 6 = 0$ are α and β.

Find an equation with integer coefficients whose roots are α^2 and β^2 [6]

4 **(a)** Express the following in the form $a + b\text{i}$ where a and b are real numbers.

(i) $(3 - 2\text{i})(2 + \text{i})$ [2]

(ii) $\dfrac{3 - 2\text{i}}{2 - \text{i}}$ [3]

(iii) $\sqrt{16\text{i}}$ [7]

(b) $z = 1 - \text{i}$ is a root of the equation

$$z^3 - 5z^2 + az + b = 0 \quad \text{where } a \text{ and } b \text{ are integers.}$$

Find the other two roots and the values of a and b. [7]

5 **(a)** A complex number is given by $z_1 = -\sqrt{3} + 3\text{i}$

Find:

(i) the modulus of z_1 [2]

(ii) the argument of z_1 in radians. [2]

The complex number z_2 has modulus $\sqrt{3}$ and argument $\dfrac{\pi}{4}$

Find in modulus argument form:

(iii) $z_1 z_2$ [4]

(iv) $\dfrac{z_1}{z_2}$ [4]

(b) The complex number z satisfies both $|z - 3\text{i}| \leqslant 3$ and $\dfrac{\pi}{2} \leqslant \arg z \leqslant \dfrac{2\pi}{3}$

Sketch on an Argand diagram the region satisfied by z. [6]

6 The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} a & 4 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

(i) Find the determinant of **M** [4]

(ii) Find the value of a for which the matrix equation

$$\mathbf{M}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$$

does not have a unique solution. [2]

(iii) Find the general solution of the system of equations

$$\begin{aligned} 3x + 4y + z &= 6 \\ x + 2y - z &= 0 \\ x + y + z &= 3 \end{aligned}$$

[4]

7 **(a)** The lines l_1 and l_2 have equations

$$l_1: \quad \mathbf{r} = 3\mathbf{i} + \mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$$
$$l_2: \quad \mathbf{r} = 2\mathbf{i} + 5\mathbf{j} + \mu(\mathbf{i} - \mathbf{j} + \mathbf{k})$$

(i) Show that the lines l_1 and l_2 intersect. [6]

(ii) Find the coordinates of the point of intersection. [1]

(b) Two planes are given by

$$\mathbf{r}\,.\,(\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = -5$$
$$\mathbf{r}\,.\,(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = 15$$

(i) Find, in vector form, an equation of the line of intersection of the planes. [6]

(ii) Find, in degrees, the acute angle between the planes. [5]

THIS IS THE END OF THE QUESTION PAPER
