GCE

Rewarding Learning

Chief Examiner's Report

Mathematics

Summer Series 2019

Foreword

This booklet outlines the performance of candidates in all aspects of this specification for the Summer 2019 series.

CCEA hopes that the Chief Examiner's and/or Principal Moderator's report(s) will be viewed as a helpful and constructive medium to further support teachers and the learning process.

This booklet forms part of the suite of support materials for the specification. Further materials are available from the specification's section on our website at www.ccea.org.uk.

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GCE MATHEMATICS

Chief Examiner's Report

Subject Overview

As this was the first assessment of a new specification, it was pleasing to see the high quality of work produced by the candidates. Teachers and candidates are to be commended for their diligent preparation for this suite of papers.

All papers provided opportunities for discrimination between candidates, with the early questions in each paper being accessible to all. The increased focus on problem-solving was evident in the later questions on each paper and provided challenge for the most able candidates.

There was evidence of the use of more sophisticated calculators. Whilst this is to be encouraged, it is important that candidates provide sufficient evidence of the methods employed to ensure that they are awarded full marks.

Candidates should be reminded to read each question carefully and ensure that they provide the answer as required.

Assessment Unit AS 1 Pure Mathematics

Unit Overview

The paper was very accessible, with the vast majority coping well with the first 5 questions.

Less able candidates had the opportunity to respond positively, with almost all getting full marks in Question 1 (a). Questions 6, 7, 8 and 9 enabled the stronger candidates to show their mathematical ability and problem-solving skills.

Candidates should take care with use of calculators and ensure that they show adequate method in the full development of their solutions.

Candidates appeared to have enough time to complete the paper.

Q1 **(a)** This was a standard first question which was very well answered. Those ending up with incorrect solutions generally made a new attempt and corrected previous errors.

(b) **(i)** This was poorly attempted with a small number of candidates finding the vector but not its magnitude. Many struggled to find the vector AB.

(ii) This was surprisingly weak with a large number of candidates not knowing how to find the angle between a vector and x-axis in 2 dimensions.

Q2 **(a)** This was very well attempted with the vast majority of candidates getting full marks.

(b) This part was generally well attempted but some candidates did not state the equation of the asymptote in the required form.

Q3 **(a)** **(i)** Almost all candidates scored full marks in this part.

(ii) This was also well attempted.

(b) Some candidates attempted to take ln at the first line of working and then ran into difficulties. A significant number of candidates do not appear to understand the concept of an exact solution.

Q4 **(a)** This was attempted very well with most candidates scoring full marks. However, presentation of solutions was generally poor when solving the quadratic inequality with incorrect mathematical notation used.

(b) This part was also well attempted with most candidates using completing the square, though a sizeable number compared with the general form of the circle equation.

A surprising number found the correct radius but did not find the area of the circle.

Q5 **(a)** This was generally very well attempted with the vast majority of candidates integrating correctly and substituting in the limits correctly. Quite a few however failed to solve correctly for k.

(b) Again, this part was attempted well with only a small number of candidates not changing into a quadratic in sine. However, having found the correct values for sin θ, far too many failed to gain the last two marks both by giving incorrect angles and/or adding in incorrect solutions.

Q6 **(a)** **(i)** A number of candidates treated the tin as a closed cylinder and some did not know the formula for the curved surface area. Algebraic manipulation was disappointingly weak.

(ii) The vast majority of candidates got the first two marks by substituting their expression for h into the correct volume formula.

(b) Generally very well attempted with only a very small number of candidates not knowing to differentiate or differentiating incorrectly. Quite a few candidates gave the radius as ± 10, not rejecting the impossible negative value.

Q7 This question was very well attempted with the vast majority of candidates scoring full marks. Most used the factor theorem to find the factor $x - 2$, with a small number dividing the cubic by different possible factors until they got a remainder of zero. Having factorised the cubic, almost all went on to get the correct solutions.

There was evidence in this question of use of calculators with only the solutions being given. Since this question is designed to assess the use of the factor theorem it is essential to show clear development of the solution if full marks are to be awarded.

Q8 **(i)** This part caused a lot of problems with only the better candidates simplifying their expression for the gradient of AB. This led to difficulty in stating a correct expression for the perpendicular to AB, with many unable to correctly state the negative reciprocal. Many did get the method marks but if the original gradient had not been simplified then few managed to get the correct coordinates of P.

(ii) This was much better attempted with many candidates getting full marks. Too many however failed to attempt the question, possibly as a result of struggling so much with Part (i).

Q9 **(i)** This was very poorly attempted with most candidates failing to use triangle OBD to find x in terms of r and cos θ. Of those who applied the cosine rule correctly to triangle AOD, very few were able to reduce cos(180 – θ) to – cos θ.

(ii) Many candidates did not attempt this part. Candidates, more often than not, tried to work with the expression for AD but made little progress. Only the best candidates were able to link to the right-angled triangle to establish cos $\theta = \frac{1}{2}$ and then substitute to get k.

This question was very effective in developing and assessing the problem-solving skills of the candidates.

Assessment Unit AS 2 Applied Mathematics

Unit Overview

In this unit candidates had the opportunity to show their knowledge and understanding of both Mechanics and Statistics. The paper adequately addressed the topics on the specification and provided candidates ample opportunity to display their abilities. The paper was well differentiated and gave candidates of varying abilities access to marks throughout the paper.

Since the paper progresses in difficulty within each section candidates should take care with time management and allocate equal time to each section.

Q1 The majority of candidates performed well in this question. It provided an opportunity for candidates to correctly resolve forces and to find a resultant. The most common error was that candidates did not correctly show or state the direction and so a number lost the final mark for this question. A small number of candidates were not sure how to approach this question and others made minor/careless arithmetic errors.

Q2 This question was well answered by the majority of candidates. A small number of candidates used an incorrect equation of motion in Part (ii) with a greater number of candidates failing to complete the question by finding the magnitude of their displacement vector. Some small numerical errors were present highlighting the importance of accuracy and careful reading of the values listed in the question.

Q3 **(i)** This part of the question was not well answered as a number of candidates failed to give enough detail in order to gain the mark.

(ii) This caused difficulty for candidates who were not able to accurately find the area below the graph in order to find time T. The majority of candidates did know how to approach this question but struggled with the numerical and algebraic aspects within the question.

Q4 **(i)** A small number of candidates included extra forces on the particles. Candidates should note the importance of arrows and labels on all forces.

(ii) This part showed that candidates were familiar with the mechanical principles necessary to solve this type of problem. Common errors included: direction of forces, rounding issues and the incorrect solution of the various equations.

(iii) A large number of candidates did not manage to gain any marks. They failed to recognise the problem posed when one block hits the ground. It was important that candidates attempted to calculate a new acceleration and apply this properly to deduce whether the horizontal block would strike the table. This question was a good discriminator for the top candidates.

Q5 Generally, this question was well attempted by candidates.

(a) Unfortunately, the definition of a census often lacked detail and clear understanding.

(b) This provided a suitable opportunity for candidates to accurately display their understanding of sampling bias in a contextual question.

Q6 Question 6 should have given candidates the opportunity to display their understanding of some basic statistical concepts. However, this question was not well answered.

(a) **(i)** Most candidates were able to successfully find the mean, but a large number struggled to correctly find the standard deviation. A small number provided the best estimate for a population based on a sample which was not required here. Candidates should be encouraged to use their calculator here, clearly showing the descriptive statistics required to calculate their answer.

(ii) This demonstrated a lack of understanding of how to calculate quartiles from listed data. A small number of candidates did not correctly follow the definition of an outlier given in the question and instead attempted other possible outlier descriptions - this did not achieve any marks.

(b) **(i)** This part was well answered by most candidates, showing clear understanding of histograms and their use.

(ii) However, this part discriminated between stronger and weaker candidates with a number not able to find the appropriate frequencies. A small number of candidates did correctly identify the required frequency but failed to leave their answer as a probability.

Q7 This was a new topic and most candidates made a reasonable attempt at this. Again here, it is useful to encourage candidates to make appropriate use of their calculator by clearly stating the descriptive statistics used to calculate their coefficient. A small number of candidates were unfamiliar with the methodology required whilst others were unable to accurately perform the calculations. Those candidates who chose to complete this question without the aid of their calculator would have penalised themselves in terms of the time used.

Q8 The first two parts of this question were not so well answered.

(i) Most candidates recognised the Binomial requirement in the question.

(ii) It was perhaps surprising that a large number of candidates were not able to cope correctly with the level of algebra. There was also a small number of candidates who tried to verify rather than prove the value of ‘p’ and therefore did not obtain full marks.

(iii) The final part of this question was well answered with the majority of candidates accurately displaying their knowledge of finding probabilities using the Binomial Distribution. This part again provided candidates with the opportunity to use their calculator. However, it is important candidates show their full development i.e. it was not possible to find the answer directly from their calculator or from the cumulative binomial tables. Hence, candidates who did not clearly show how they used the obtained value, to arrive at their answer, lost marks.

Assessment Unit A2 1 Pure Mathematics

Unit Overview

There was a wide range of scores achieved on this paper. However, many candidates appeared to be very well prepared and achieved excellent marks. There was no evidence of any issue with the time allocation. There was progression in the level of difficulty of questions, with the later questions providing differentiation between the candidates. It was evident that candidates were making greater use of their calculators in this paper. While this is to be welcomed and encouraged, there is still a need to show sufficient development of work within each question.

Q1 This was a routine starter question and most candidates gained full marks. However, some missed the negative sign when re-arranging.

Q2 This question was easily accessed by most candidates. The main problem was failing to simplify the algebraic expression when giving their final answer. There were some instances of mis-reading the question with a small number of candidates using a as the parameter and also a few who differentiated and found $\frac{\mathrm{d}y}{\mathrm{d}x}$ instead of the cartesian equation.

Q3 **(i)** Quite a variety of methods were used to find the angle. Some of these methods were much more difficult and increased the potential for calculation errors. However, most candidates did eventually find the correct angle in the correct form.

(ii) Most candidates managed to complete this question successfully. However, some again chose much more complex methods. Whilst these were perfectly correct and valid they did require a longer time to complete. Candidates should avoid over-rounding at early stages of the question. This did create a significant impact on the final answer and was therefore penalised in the final mark.

Q4 **(i)** This question differentiated between the candidates, with the most able proving the identity in a few lines as shown on the markscheme. The key to quick completion was to recognise that $\cot\theta = \frac{\cos\theta}{\sin\theta}$. Others worked through much less efficient methods with a good number eventually reaching a correct conclusion. However, a few became entangled in the more complex algebra and were unable to proceed.

(ii) Most candidates answered this correctly, but some were unable to recognise the connection with Part (i) and simply typed $\frac{\pi}{8}$ into their calculation, thus producing a decimal value.

Q5 **(a)** **(i)** Almost all candidates understood how to find the range, but a number did not use correct function notation.

(ii) Most candidates knew how to find the inverse function, but many lost the final mark as a result of incorrect notation or an incorrect domain.

(iii) This was generally answered well, although a mark was often lost because of missing or incorrect labels on each axis.

(iv) This was very well answered.

(b) **(i)** This was also well answered, although a few candidates failed to label their diagram correctly.

(ii) This was more challenging than Part (i), but again was generally well answered.

Q6 **(i)** This was a standard trigonometry question and did not pose any great problem for most candidates. However, a few did not show full working and tried to take shortcuts to the final solution.

(ii) This question differentiated between candidates. Whilst many completed the problem without any difficulty, others did not realise that they needed to find the minimum value of the sine term in the denominator and were therefore unable to make any further progress.

Q7 **(i)** Many candidates successfully completed this question and were awarded full marks. However, a surprising number failed to find the correct value of h and lost a significant proportion of the marks.

(ii) Since this question involved an improper fraction, it very clearly differentiated between candidates. Those who recognised the improper fraction and dealt with it correctly usually progressed to gain full marks. Occasionally a negative sign was missed in the long division. Those who ignored the improper fraction lost a significant number of marks.

(iii) Almost all candidates answered this correctly.

Q8 **(i)** Most candidates were able to separate the variables and integrate correctly, although some omitted the constant of integration. The final rearrangement of terms was not always fully developed and it was sometimes doubtful if candidates would have obtained the correct final answer if this had not been a "show that" problem.

(ii) This was well answered, although some candidates did not work with P_0 and $2P_0$ but instead substituted numerical values.

(iii) This was also well answered, again with some candidates using numerical values as in Part (ii).

(iv) Candidates struggled to give a clear and concise contextual limitation of the model.

Q9 **(i) & (ii)** posed no difficulty for most candidates with many gaining full marks.

(iii) Only the best candidates recognised which functions were required for the Newton-Raphson method. Most were only able to score 1 mark out of 5.

Q10 **(a)** The majority of candidates realised to use integration by parts and correctly set $u = \ln x$.

They were then able to easily complete the integration. Some, however, set $\frac{dv}{dx} = \ln x$ and were therefore unable to make further progress with their integration.

(**b**) This was a more challenging question. Some candidates re-wrote the stated substitution as $u = x + 2$ and got into difficulty with their substitution. Errors included differentiating incorrectly and substituting incorrectly, especially omitting the dx term.

These mistakes often led to complex algebra which was susceptible to further errors. It was evident that calculators were being used in some cases to provide the final solution. Whilst this is acceptable as a checking mechanism, the question clearly stated that the method of substitution was required and therefore full working of this method was essential.

Q11 (i) It was perhaps surprising that this question was so poorly answered. Many candidates were able to do little other than set $\sin 2x = \cos 2x$. Others tried the double angle formula for $\tan 2x$ and created a much more complex problem. A number substituted the given values and simply verified that they were correct. This is not acceptable practice in a "show that" question.

(ii) This question was not well answered. Many candidates did not appear to know the standard method of finding the area between 2 curves. Instead, they divided the region up into a number of small sections and tried to find each of these areas, with some being more successful than others in using this procedure. Those who used the standard method had no difficulty in completing the question.

(iii) This was a standard volume of revolution question and was well answered. Only a few errors were seen and these included squaring $(4 - x^2)$ or using the wrong limits.

Q12 (a) (i) This proof was poorly answered. Some candidates tried to prove the formula in terms of a, n and d and then convert it back in terms of a, n and l in the final stage. Only a few were completely successful. Others had no real idea of how to approach the problem and made little, if any, progress.

(ii) & (iii) were both routine questions and were extremely well answered.

(b) (i) Almost all candidates were able to complete this question.

(ii) Success in this part often depended on how candidates had approached Part (i). For many, they had not spotted that this was a GP and therefore could not make any further progress. For those who had made this connection they were able to easily produce the required result.

(iii) Most candidates were able to gain all marks in this question. However, some only used trial and improvement, rather than an algebraic solution.

Assessment Unit A2 2 Applied Mathematics

Unit Overview

As this was a new specification, it was pleasing to see that, in general, the candidates responded well to the questions in this paper. It was evident that many candidates were well prepared for this exam and knew the specification well. The questions were all well attempted and there were many candidates achieving very high marks.

The paper was accessible to all. Most candidates were able to understand what was expected of them and a large number of candidates produced accurate responses. The stronger candidates were given the opportunity to demonstrate their knowledge and understanding of each topic; they were challenged to think beyond their basic understanding in each question thus checking their full understanding of the topic that was being tested. In particular they were challenged to show the depth of their knowledge through reasoning and thought. The questions, as they progressed, thoroughly differentiated between candidates, and there was good differentiation between very good candidates and excellent candidates.

Q1 **(i)** This was very well answered. However, there were a few candidates who were unaware that there should have been two different tensions on the diagram.

(ii) This question was well answered, with moments being correctly chosen by almost everyone. There were a number of candidates who opted to take moments twice, ignoring the vertical components. This may have been a slightly more difficult strategy where it was easier to make mistakes, but candidates who chose this option handled it well.

Q2 **(i)** Momentum before = momentum after was seen in nearly every script. Very few candidates made errors with directions and fewer again made errors with organising their 'ku' term. This was generally answered correctly.

(ii) The majority of candidates answered this well, including directions and choosing the correct mass to begin with. A few made errors by omitting 'mu' or 'u'.

Q3 Overall this question was very well answered. Candidates are clearly well rehearsed in these types of questions.

(i) & (ii) Nearly all candidates got full marks here. Factorising and differentiation were both very good.

(iii) This question is clearly well practised. There were very few candidates who left out or made no reference to c. The majority managed to fill in all three times with t = 0.2, 4 and 5. When answered incorrectly, it was still the case that candidates attempted to find directions or knew to check their own directions and so they achieved a good proportion of the marks. There were very few who made calculation errors. There were only a small number of candidates who did not use the correct times, and only put in t = 5, but they still managed to gain the first four marks. A number of candidates chose to integrate between the correct limits, which was also very well done. One or two were substituting in the time between each change of direction and so misunderstood the concept of the topic.

Q4 **(i)** This first part was generally well answered. A few candidates could not find the correct t to substitute in but made attempts and managed to achieve some marks if they chose the correct equation of motion. A few candidates lost all marks here as they were unaware of how to start this question. Although some of these candidates managed to obtain $u\sin\theta$ and $u\cos\theta$ correctly, they did not have sufficient understanding of the motion to gain any marks.

(ii) The word 'hence' was missed by a number of candidates who tried different methods of trying to find the correct angles. Some obtained these angles but did not gain full marks since they had not followed the instructions stated in the question. There were a number of candidates who did not find Part (i) but were still able to achieve full marks here for substituting in the desired values. Surprisingly, some candidates seemed to have no understanding of projectile motion at all.

Q5 For the most part, it was the layout of these answers that was a little concerning. While many candidates managed to achieve the complete correct answers, there were a number who only considered moments. Candidates who started with vertical components, then horizontal and then moments (and who laid this out properly on the paper), generally answered much better. Candidates should be encouraged to present their work coherently, as there were a lot of easy marks in this question and it was sometimes unclear what the candidate was doing. The use of correct trig values was excellent. 'g' was occasionally left out and therefore candidates lost a few marks as a result of a very simple error.

Q6 **(a)** **(i)** When candidates remembered to use a Venn diagram, this was answered well. Otherwise solutions were not very successful.

(ii) The majority of candidates remembered conditional probability and managed to achieve the method mark even if they had incorrect values from Part (i). If Part (i) was answered correctly this too was often answered correctly.

(iii) This is where candidates who really understood statistics were able to shine. Quite a number of candidates answered this correctly with appropriate calculations and correct reasoning. Candidates need to ensure that when they state the answer they include a final conclusion as to independent or not; they cannot leave it open for debate. It can be as simple as i.e. independent, or therefore independent.

(b) **(i)** This was probably the best answered question on the paper. It was very well done.

(ii) The majority of candidates answered this correctly; only a few were unsure of how to put WW and BW together. There were only a very small number of candidates who did not understand the question asked.

(iii) Again, if Parts (i) and (ii) were answered correctly then so was this part. Conditional probability was well understood. A number of candidates did not get this correct but managed to get some marks for method and follow through.

Q7 **(a)** **(i)** The formula for standardising was correctly used and values added in correctly. However, a number of candidates found incorrect z scores, or put in 0.674 without consideration of the minus sign. They were able to achieve an extra method mark if simultaneous equations were attempted but nothing further. On the odd occasion the variance was neglected and candidates lost the final mark.

(ii) This was well done, although too many candidates are over-rounding early in statistics questions. They must be aware that at least 4 sig figs are required throughout a question. If candidates rounded too soon for this question they lost their final mark. There needs to be greater accuracy. The very best candidates are working correctly and accurately. When candidates answer this question through calculator means, it is important that they show sufficient method in order to be awarded full marks. They should state their upper limit, their mean and standard deviation values. A diagram would be also be beneficial to explain the subtraction obtained from the

calculator.

(b) The majority of candidates seemed unfamiliar with the concept that a normal distribution follows many natural distributions. Many weird and unrequired answers were written here such as you can work out the mean and standard deviation easily.

Q8 **(i)** This question was really well answered. As a follow on from AS Level it was well thought through and answered correctly.

(ii) Where this question was attempted, it was very well executed. There were some candidates who left out the set-up of the hypotheses, but the general understanding of the question was very good. There were occasions, like all Binomials, where candidates included 7 or performed only 7 and so marks were lost accordingly. The majority who answered this question correctly also had laid it out perfectly, showing a great understanding of what was happening. The insufficient evidence to reject H_0 was worded very well. Candidates were clearly well-rehearsed in this topic when it was attempted. The tables or calculator were used correctly, and it was well presented. Only a very small number of candidates chose the incorrect conclusion or did not relate it back to the question asked.

(iii) Increasing sample size was the most common answer, though some chose decreasing the significant level which was also accepted.

Q9 **(i)** This was another question that, when answered, was performed very well. There were a few candidates who pursued a one tail test and lost marks, but the majority knew to compare to 1.645 or 0.05 depending on their method. Almost all candidates found the correct –2.556 comparison score, with only a very small number failing to use a sample. Again, there were few mistakes on the understanding of rejection or acceptance; the majority answered this correctly with only some making mistakes.

(ii) Many reasons were given; some mentioned December and the holidays so potentially a different standard or type of runner in December. This was often awarded a mark, even though explanations can be a bit worrying regarding the level of understanding. However, the very best candidates answered this perfectly.

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